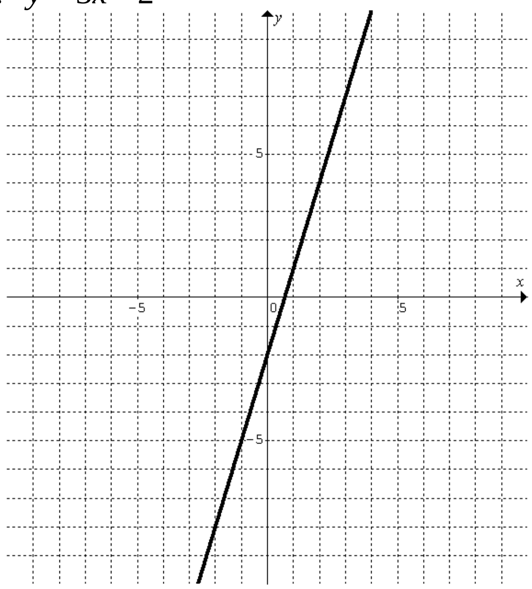
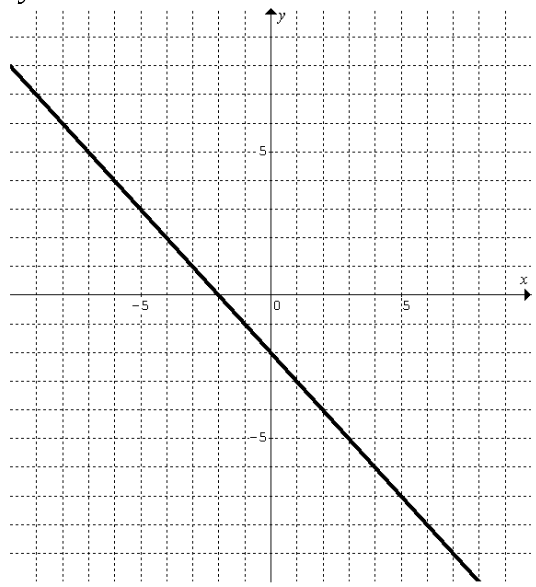
3.1 Characteristics Polynomial Functions – End Behaviour (Day 1)

For each function, identify the coefficient and the degree of the leading term. Indicate the quadrants in which each graph begins and ends.

1. $y = 3x - 2$

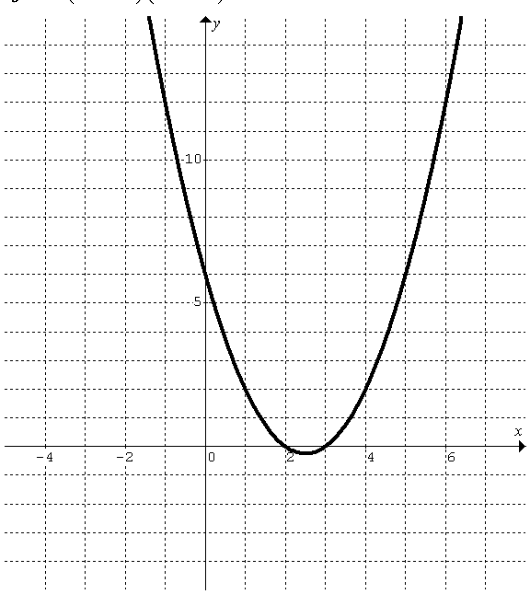


2. $y = -x - 2$



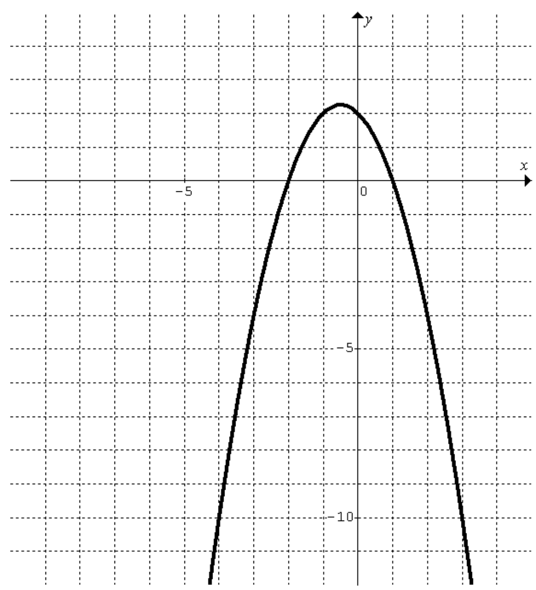
3. $y = x^2 - 5x + 6$

$y = (x-3)(x-2)$



4. $y = -x^2 - x + 2$

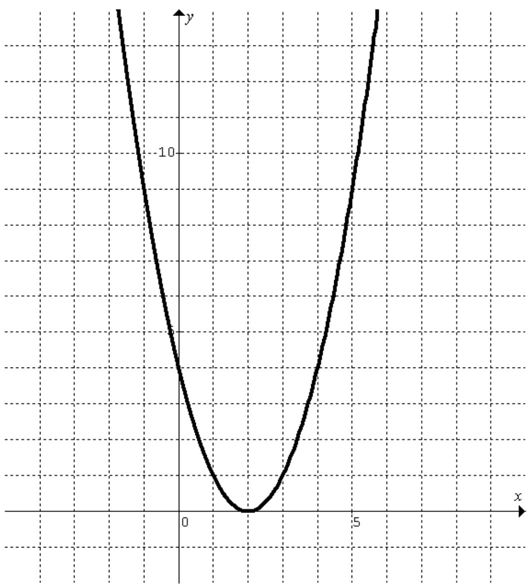
$y = -(x-1)(x+2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
1.				
2.				
3.				
4.				

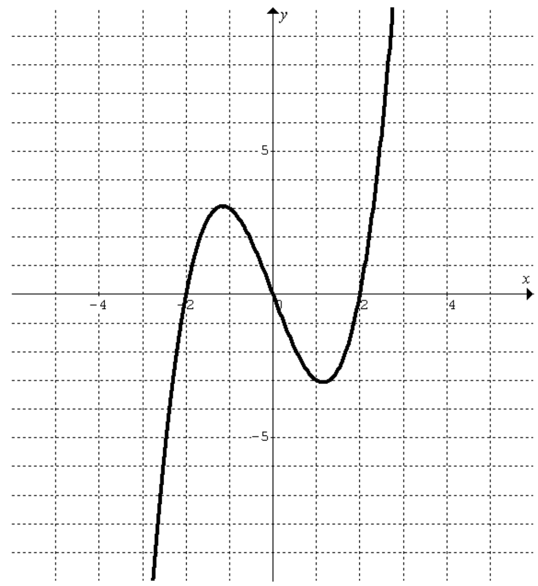
5. $y = x^2 - 4x + 4$

$y = (x-2)^2$



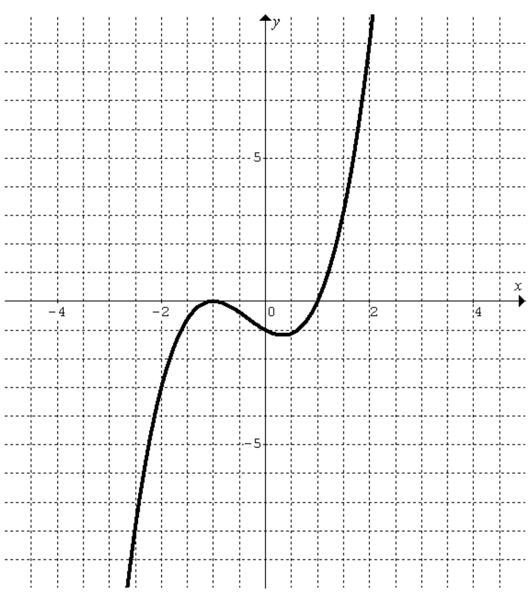
6. $y = x^3 - 4x$

$y = x(x-2)(x+2)$



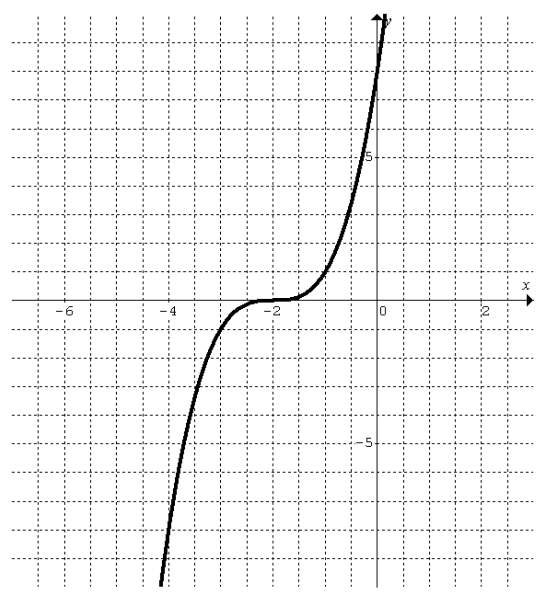
7. $y = x^3 + x^2 - x - 1$

$y = (x+1)^2(x-1)$



8. $y = x^3 + 6x^2 + 12x + 8$

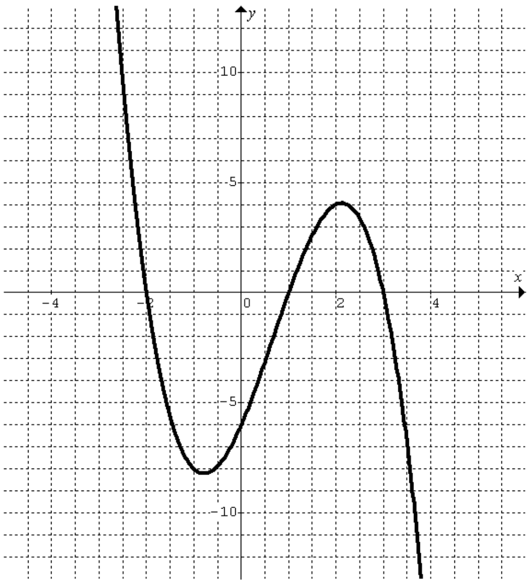
$y = (x+2)^3$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
5.				
6.				
7.				
8.				

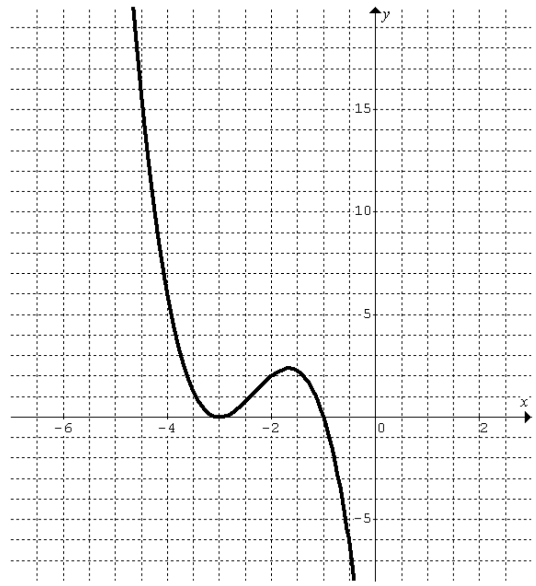
9. $y = -x^3 + 2x^2 + 5x - 6$

$y = -(x+2)(x-1)(x-3)$



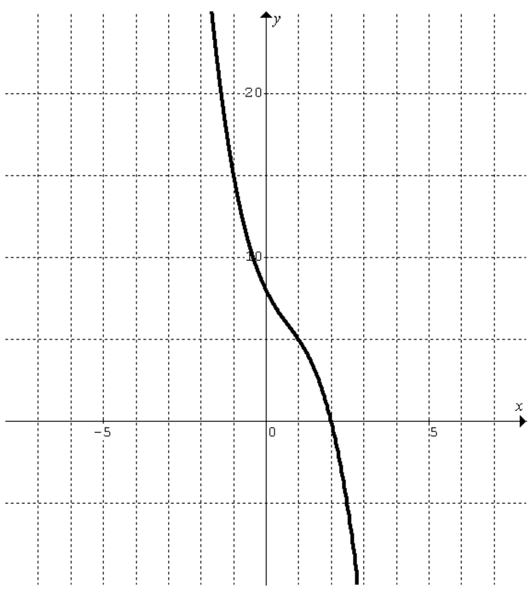
10. $y = -2x^3 - 14x^2 - 30x - 18$

$y = -2(x+1)(x+3)^2$



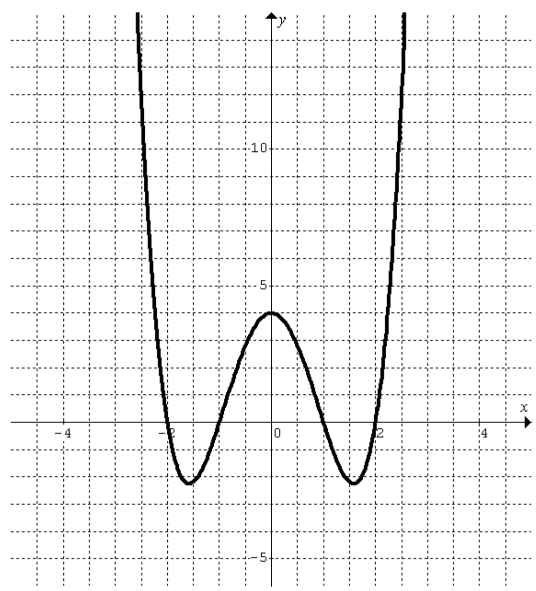
11. $y = -x^3 + 2x^2 - 4x + 8$

$y = -(x^2+4)(x-2)$



12. $y = x^4 - 5x^2 + 4$

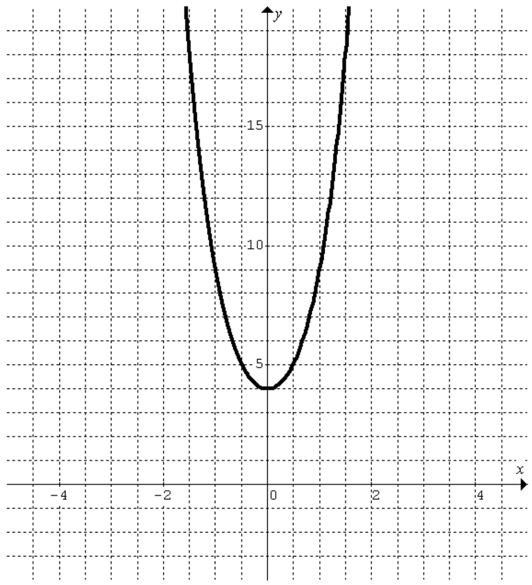
$y = (x+1)(x-1)(x+2)(x-2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
9.				
10.				
11.				
12.				

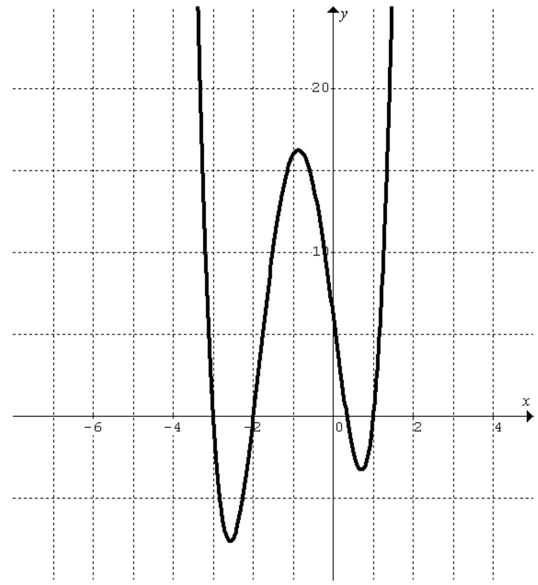
13. $y = x^4 + 4x^2 + 4$

$y = (x^2 + 2)^2$



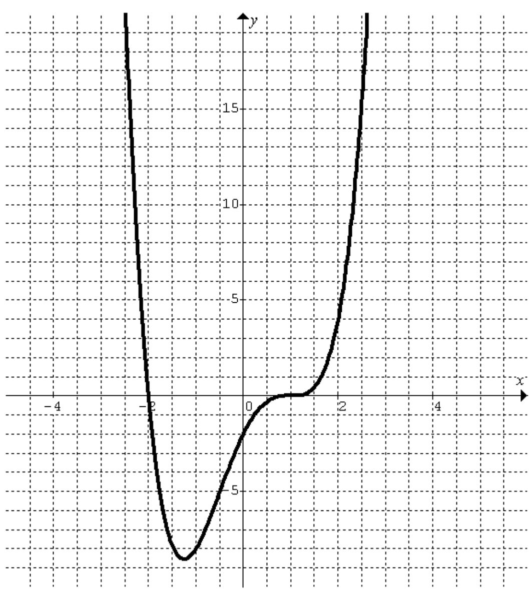
14. $y = 3x^4 + 11x^3 - x^2 - 19x + 6$

$y = (x+2)(x+3)(3x-1)(x-1)$



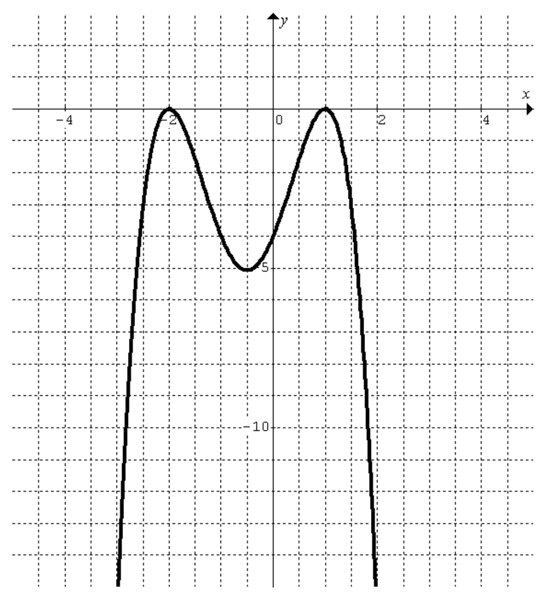
15. $y = x^4 - x^3 - 3x^2 + 5x - 2$

$y = (x+2)(x-1)^3$



16. $y = -x^4 - 2x^3 + 3x^2 + 4x - 4$

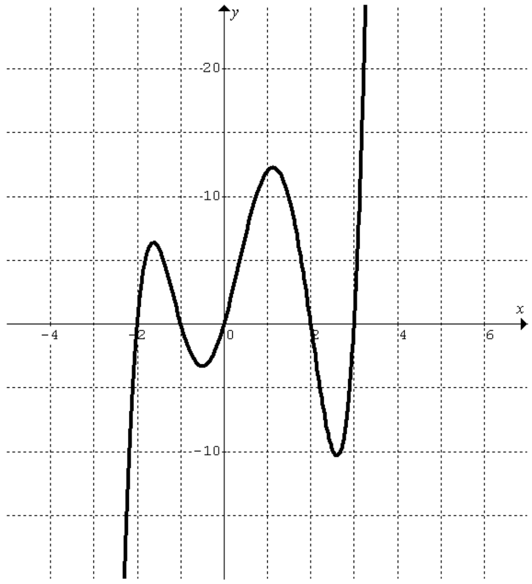
$y = -(x+2)^2(x-1)^2$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
13.				
14.				
15.				
16.				

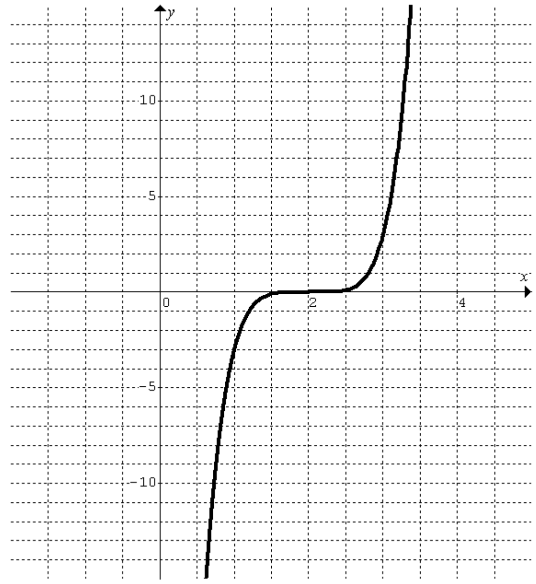
17. $y = x^5 - 2x^4 - 7x^3 + 8x^2 + 12x$

$y = x(x+1)(x+2)(x-2)(x-3)$



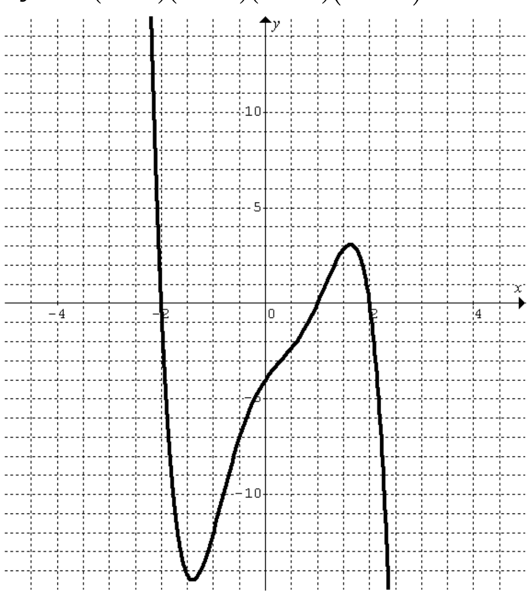
18. $y = 3x^5 - 30x^4 + 120x^3 - 240x^2 + 240x - 96$

$y = 3(x-2)^5$



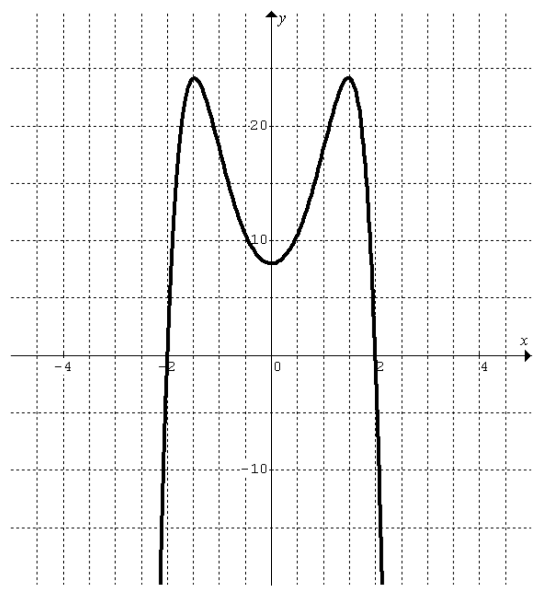
19. $y = -x^5 + x^4 + 3x^3 - 3x^2 + 4x - 4$

$y = -(x-1)(x-2)(x+2)(x^2+1)$



20. $y = -x^6 + x^4 + 10x^3 + 8$

$y = -(x-2)(x+2)(x^2+1)(x^2+2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
17.				
18.				
19.				
20.				

Observations:

1. Look at the leading term for each function. Which functions have an odd degree and a positive leading coefficient?

2. Based on your observations in question 1, the graph of a polynomial function with an odd degree and a positive leading coefficient will begin in quadrant ____ and end in quadrant ____.

3. Which functions have an odd degree and a negative leading coefficient?

4. Based on your observations in question 3, the graph of a polynomial function with an odd degree and a negative leading coefficient will begin in quadrant ____ and end in quadrant ____.

5. Which functions have an even degree and a positive leading coefficient?

6. Based on your observations in question 5, the graph of a polynomial function with an even degree and a positive leading coefficient will begin in quadrant ____ and end in quadrant ____.

7. Which functions have an even degree and a negative leading coefficient?

8. Based on your observations in question 7, the graph of a polynomial function with an even degree and a negative leading coefficient will begin in quadrant ____ and end in quadrant ____.

9. What is the domain of all of these functions?

A **relationship** exists between the number of possible *x*-intercepts and the degree of the polynomial function.

- **Odd degree**: Minimum of one *x*-intercept; maximum number of *x*-intercepts is indicated by the degree of the polynomial function
- **Even degree**: Minimum of zero *x*-intercepts; maximum number of *x*-intercepts is indicated by the degree of the polynomial function

Example: A polynomial function of degree of 6 may have no *x*-intercepts or as many as 6 *x*-intercepts.
A polynomial function of degree of 5 may have one *x*-intercept or as many as 5 *x*-intercepts.

Assignment Pg 114 #3

3.1 Characteristics of Polynomial Functions (Day 2)

- The **degree of a polynomial function** is the greatest exponent of the variable *x* that exists in the equation of the function.
- The **coefficient** of this greatest power of *x* is the **leading coefficient**. The text book calls this term a_n.
- The **constant term** represents the $y-$intercept of the function. The text book calls this term a_0.
- In PC30 the coefficients are restricted to being **integral values**, that is, they are integers.

Identify Polynomial Functions

Which functions are polynomials? State the degree, leading coefficient and the constant term of each polynomial function.

a) $g(x) = \sqrt{x} + 5$

b) $f(x) = 3x^4$

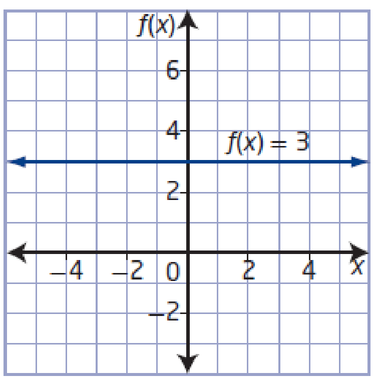
c) $y = |x|$

d) $y = 2x^3 + 3x^2 - 4x - 1$

Degree 0: Constant Function

Even degree

Number of *x*-intercepts: 0 (for $f(x) \neq 0$)



Example: $f(x) = 3$

End behaviour: extends horizontally

Domain: $\{x \mid x \in R\}$

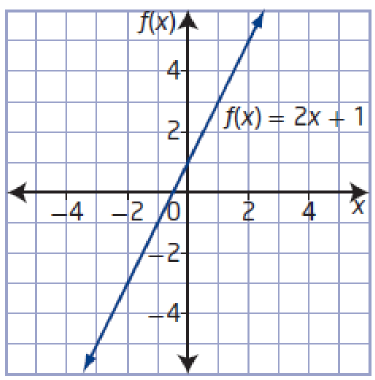
Range: $\{3\}$

Number of *x*-intercepts: 0

Degree 1: Linear Function

Odd degree

Number of *x*-intercepts: 1



Example: $f(x) = 2x + 1$

End behaviour: line extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

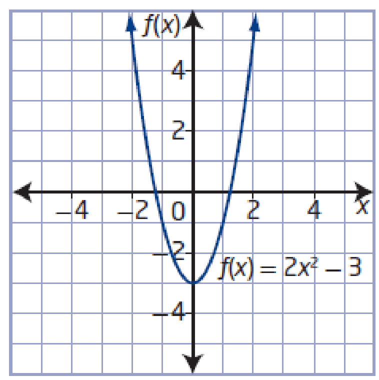
Range: $\{y \mid y \in R\}$

Number of *x*-intercepts: 1

Degree 2: Quadratic Function

Even degree

Number of *x*-intercepts: 0, 1, or 2



Example: $f(x) = 2x^2 - 3$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -2, y \in R\}$

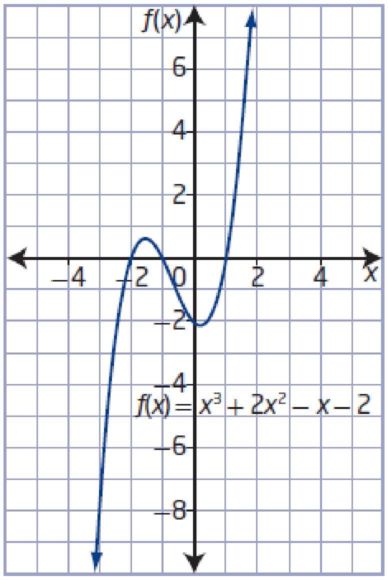
Number of *x*-intercepts: 2

What would these polynomial functions look like if the leading coefficient was less than 0?

Degree 3: Cubic Function

Odd degree

Number of *x*-intercepts: 1, 2, or 3



Example:

$f(x) = x^3 + 2x^2 - x - 2$

End behaviour: curve extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

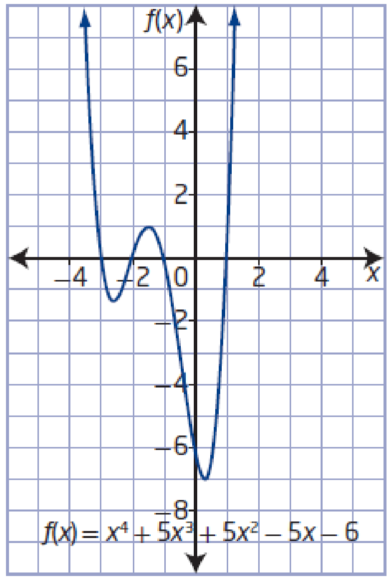
Range: $\{y \mid y \in R\}$

Number of *x*-intercepts: 3

Degree 4: Quartic Function

Even degree

Number of *x*-intercepts: 0, 1, 2, 3, or 4



Example:

$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in R\}$

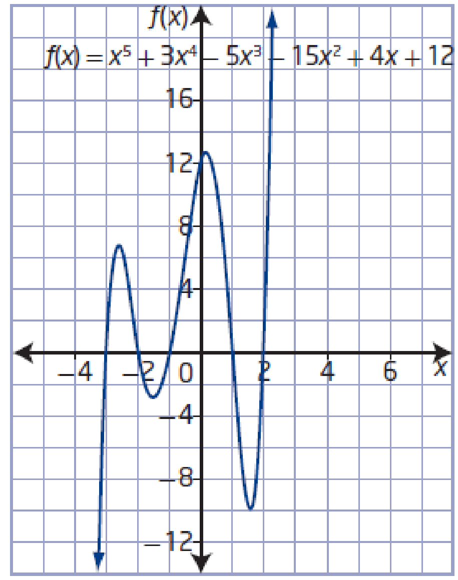
Range: $\{y \mid y \geq -6.91, y \in R\}$

Number of *x*-intercepts: 4

Degree 5: Quintic Function

Odd degree

Number of *x*-intercepts: 1, 2, 3, 4, or 5



Example:

$f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

End behaviour: curve extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \in R\}$

Number of *x*-intercepts: 5

What would these polynomial functions look like if the leading coefficient was less than 0?

Match each function to its corresponding graph:

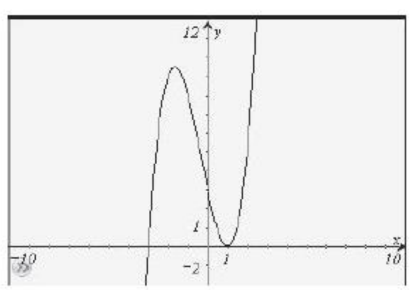
a) $g(x) = -x^4 + 10x^2 + 5x - 4$

b) $f(x) = x^3 + x^2 - 5x + 3$

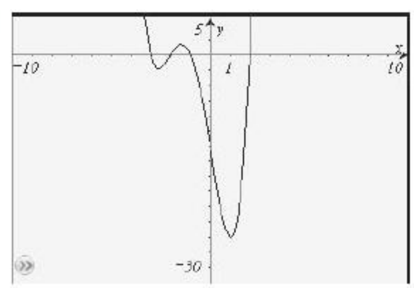
c) $p(x) = -2x^5 + 5x^3 - x$

d) $h(x) = x^4 + 4x^3 - x^2 - 16x - 12$

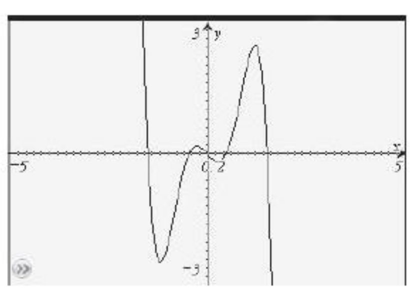
A



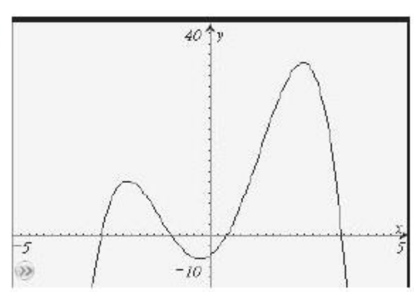
B



C



D



Application of a Polynomial Function

A bank vault is in the shape of a rectangular prism. Its volume, V, is related to the width, w, in meters, of the doorway by the function $V(w) = w^3 + 13w^2 + 54w + 72$.

a) What is the volume, in cubic meters, of the vault if the door is 1 m wide?

b) What is the least volume of the vault? What is the width of the door for this volume? Why is this situation not realistic?

<u>**Key Ideas**</u>

To remember the end behavior of polynomial functions, you may choose to only think of two cases: the line and the parabola.

Odd degree "the line".

- Positive L.C.: think of a line with positive slope.
- Negative L.C.: think of a line with negative slope.

Even degree **"**the parabola".

- Positive L.C.: think of a parabola $y = 1x^2$.
- Negative L.C: think of a parabola $y = -1x^2$

Assignment Pg 114 #1, 2, 6, 7ab, 9

3.2 The Remainder Theorem

Synthetic division can always use when dividing a polynomial by a binomial, $x-a$.

In general, "a"| c o e f f i c i e n t s

____

| remainder

We write the final answer (the quotient) as... polynomial + $\frac{remainder}{x-a}$. We also note restrictions on the variable due to the fact that we are dividing by $x-a$ (x is in the denominator).

Example 1) Divide $2x^3+3x^2-4x+15$ by $x+3$.

Restrictions:

Example 2) Use synthetic division to determine $\frac{x^3+7x^2-3x+4}{x-2}$.

Restrictions:

Example 3) Divide $3x^4-2x^3+4x^2-1$ by $x-2$

Restrictions:

The **remainder theorem** states that when a polynomial, $P(x)$, is divided by a binomial of the form $x-a$, the remainder is $P(a)$. Here we want to figure out what "a" is, and then substitute that value into the polynomial everywhere we see an *x*. The result we achieve is the remainder.

Example 4) Use the remainder theorem to determine the remainder when $P(x)=x^3-10x+6$ is divided by $x+4$.

- We are dividing by $x+4$. What is "a"?

Now, $P(\quad)=$

Does this result match what you get when performing synthetic division? Show me.

Example 5) Determine the value of *k* if the remainder of $(x^3+4x^2-x+k)\div(x-1)$ **if the remainder is 3.**

The unknown value of *k* (or the variable of their choice) may be anywhere as a coefficient. Don't panic. Bring it along for the ride as you perform your division!

Assignment Pg 124 #4 – 8, 10 – 12

3.3 The Factor Theorem

The factor theorem states that $x-a$ is a factor of a polynomial $P(x)$ if and only if $P(a)=0$.

Example 1) Is $(x-1)$ a factor of $P(x)=x^3-x^2-5x+2$?

Example 2) Is $(x+2)$ a factor of $P(x)=x^3-x^2-5x+2$?

Factoring Higher-Degree Polynomials

The **integral zero theorem** states that if $x-a$ is a factor of a polynomial $P(x)$ with integral coefficients, then a is a factor of the constant term of $P(x)$. **Find the factors of the constant and test them to see if you get a remainder of zero. If you do, you've found a factor!**

Example 3) Factor $2x^3+x^2-13x+6$ completely.

Example 4) Factor $x^5-5x^4+4x^3+16x^2-32x+16$ completely.

Assignment Pg 133 #2, 3, 5, 7, 11

3.4 The Zeros of a Polynomial Function (Day 1)

***y*-intercept** – the value at which the graph crosses the *y*-axis

- the value of y when $x = 0$

***x*-intercept(s)** – the value(s) at which the graph crosses the *x*-axis

- the value of x when $y = 0$
- also called the zeros of the function

Complete the chart using the functions and graphs from the examples in your notes for Section 3.1 – End Behaviour. For the **"behavior of the graph at the zero",** you have **3 options**: cross, tangent not cross (TNC), and tangent and cross (TC)

	constant term	*y*-intercept	algebraic factors of the polynomial	multiplicity	*x*-intercept(s) or zeros	behaviour of the graph at the zero
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						

	constant term	*y*-intercept	algebraic factors of the polynomial	multiplicity	*x*-intercept(s) or zeros	behaviour of the graph at the zero
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						

How can you identify the ***y*-intercept** of the function?

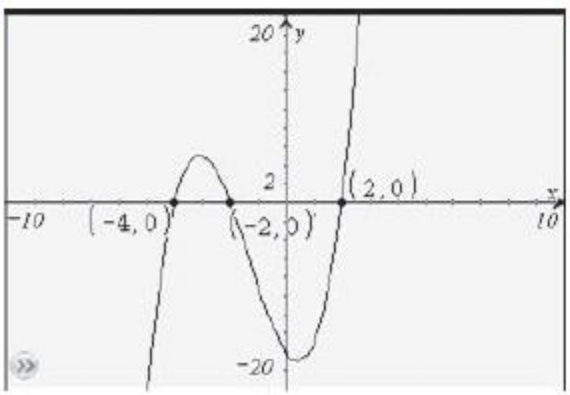
What is the **relationship** between the multiplicity of a factor and the behaviour of the graph at the corresponding zero (or *x*-intercept)?

Multiplicity of 1: **Multiplicity odd but $\neq 1$:** **Multiplicity even:**

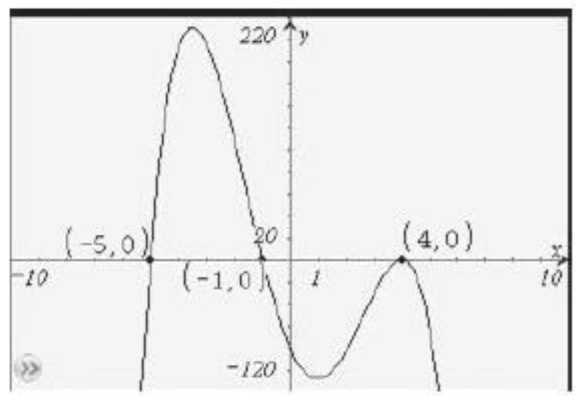
Example 1) For each graph, determine:

- The least possible degree
- The sign of the leading coefficient
- The *x*-intercepts and the factors of the function (with least possible degree)
- The intervals where the function is positive (above the *x*-axis) and intervals where it is negative (below the *x*-axis)

a)

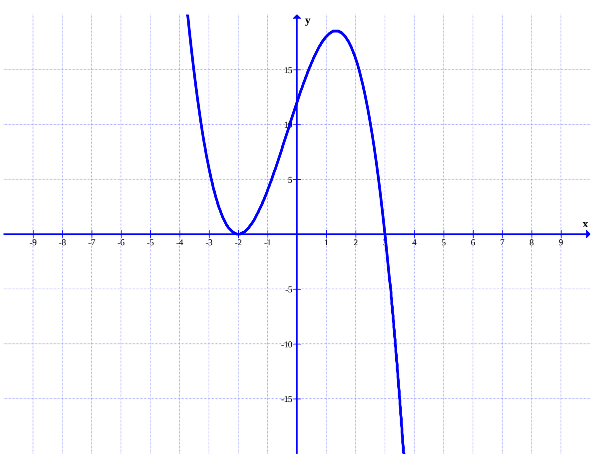


b)



Your Turn

For the following graph, determine the characteristics as listed above.



- The least possible degree
- The sign of the leading coefficient
- The *x*-intercepts and the factors of the function (with least possible degree)
- The intervals where the function is positive (above the *x*-axis) and intervals where it is negative (below the *x*-axis)

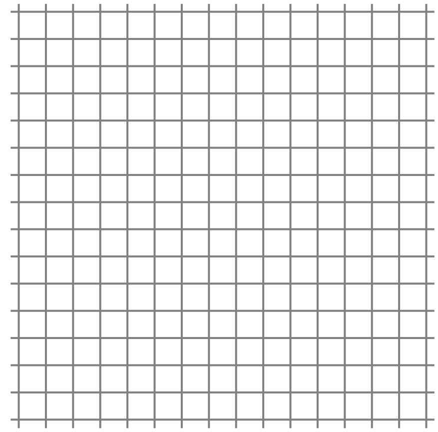
Assignment Pg 148 #4

3.4 The Zeros of a Polynomial Function (Day 2)

Sketching Graphs of Functions: Equation Already Factored

Example 1) Sketch the graph of $y=(x-1)(x+2)(x+3)$.

x-intercepts:



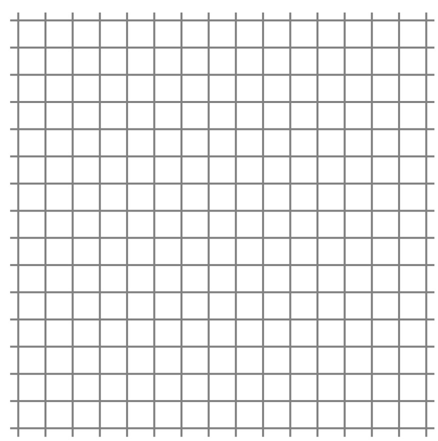
(also discuss multiplicity and behavior)

Degree:
Leading coefficient:
Begins in _______ and ends in _______
y-intercept:

'reference points' to add detail to graph:

Example 2) Sketch the graph of $f(x)=-(x+2)^3(x-4)$

x-intercepts:



(also discuss multiplicity and behavior)

Degree:
Leading coefficient:
Begins in _______and ends in _______

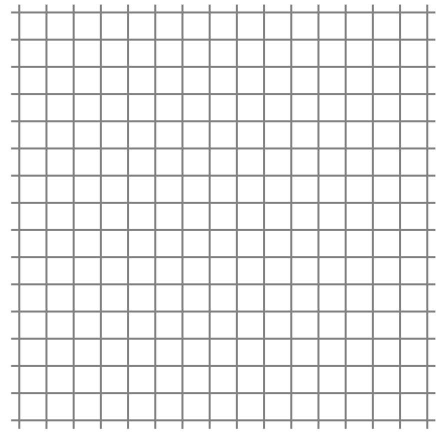
y-intercept:

'reference points' to add detail to graph:

Sketching Graphs of Functions: Equation Not Yet Factored

Example 3) Sketch the graph of $y = -2x^3 + 6x - 4$

Factor!



x-intercepts:

Degree:

Leading coefficient:

Begins in _______ and ends in _______

y-intercept:

Assignment Pg 149 #9e, then 8 (use our list of info to sketch the graphs of the equations in #7)

3.4 The Zeros of a Polynomial Function (Day 3)

Applying Transformations to Sketch a Graph

- a, b, h, k are back
- Knowing the precise shape of base functions such as $y = x^3$, $y = -x^3$, $y = x^4$ and $y = -x^4$ is **essential**
- You can use the key points of the base function and **use mapping** to find the points of the new, transformed function.

Example 1) The graph of $y = x^3$ is transformed to obtain the graph of $y = -2(4(x-1))^3 + 3$

State the parameters, use mapping to find new points and sketch the graph.

$a =$ $b =$ $h =$ $k =$

For $y = x^3$, mapping of key points looks like:

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

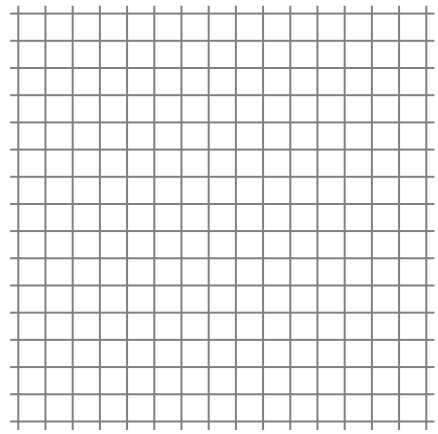
$(-2, \quad) \rightarrow$

$(-1, \quad) \rightarrow$

$(0, \quad) \rightarrow$

$(1, \quad) \rightarrow$

$(2, \quad) \rightarrow$



Word Problems

- A “Let statement” is required.
- Quadratic formula?
- A sentence is required at the end, **not a solution set**.

Bill is preparing to make an ice sculpture. He has a block of ice that is 3 ft wide, 4 ft high, and 5 ft long. Bill wants to reduce the size of the block of ice by removing the same amount from each of the three dimensions. He wants to reduce the volume of the ice block to 24 feet cubed.

a) Write a polynomial function to model this situation.

b) How much should he remove from each dimension?

Example 2) Three consecutive integers have a product of -210. What are the three integers?

Example 3) Determine the cubic equation with zeros -3 (multiplicity of 2) and 2, and y-intercept -18.

Assignment: Pg 148 #3, 5, 6 (mapping), 10, 11, 13, 14, 16