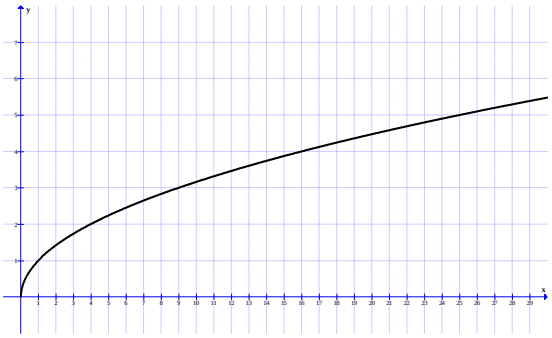
Chapter 2: Radical Functions

2.1 Radical Functions and Transformations (Day 1)

- For the function $y = \sqrt{x}$, the radicand, *x*, must be greater than or equal to zero [$x \geq 0$]. **Why**?



Consider this function to be the **base radical function.**

x-intercept of

y-intercept of

$(0,0)$ is referred to as **the end point**.

Domain: $\{x \mid x \geq 0, x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Note some of the KEY COORDINATES of this function.

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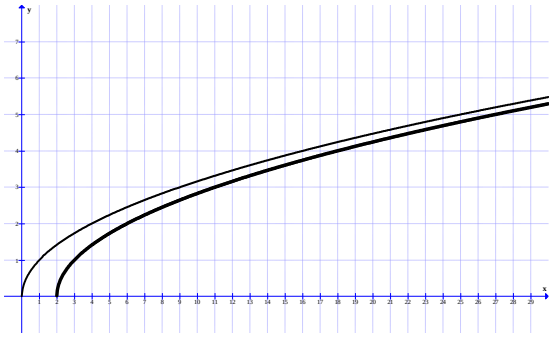
- For the function $y = \sqrt{x-2}$, *x* must be greater than or equal to 2. Algebraically show **why**.



Domain: $\{x \mid x \geq 2, x \in \mathbb{R}\}$

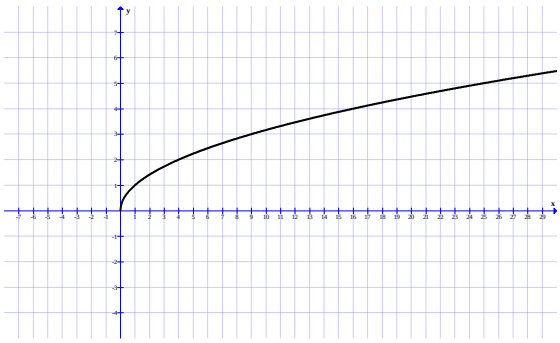
Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

- Given what you know about translations, draw the function $y = \sqrt{x} - 3$. The base function $y = \sqrt{x}$ is already on the coordinate plane for you.

Domain:

Range:

- Given what you know about translations, draw the function $y = \sqrt{x+5}$. The base function $y = \sqrt{x}$ is already on the coordinate plane for you.



Domain:

Range:

Transforming Radical Functions

The **base radical function** $y = \sqrt{x}$ is transformed by changing the parameters $a, b, h, \text{and } k$ in the equation $y = a\sqrt{b(x-h)} + k$.

a	• Vertical stretch by a factor of $\lvert a \rvert$. If a is negative [$a < 0$], the graph of $y = \sqrt{x}$ is reflected in the $x-$axis.
b	• Horizontal stretch by a factor of $\frac{1}{\lvert b \rvert}$. If b is negative [$b < 0$],the graph of $y = \sqrt{x}$ is reflected in the $y-$axis.
h	• Horizontal translation. $(x-h)$ means the graph of $y = \sqrt{x}$ moves h units **right**. For example, $y = \sqrt{x-1}$ means that the graph of $y = \sqrt{x}$ moves 1 unit right. • $(x+h)$ means the graph of $y = \sqrt{x}$ moves h units **left**. For example, $y = \sqrt{x+5}$ means that the graph of $y = \sqrt{x}$ moves 5 units left. This translation has the **opposite effect** than many people think. It is a common error to think that the + sign moves the graph to the right and the **–** sign moves the graph to the left. **This is not the case**.
k	• Vertical translation. $+k$ means the graph of $y = \sqrt{x}$ moves k units **up;** $-k$ means the graph of $y = \sqrt{x}$ moves k units **down**

Assignment: Examples 1 and 2

2.1 Radical Functions and Transformations (Day 2)

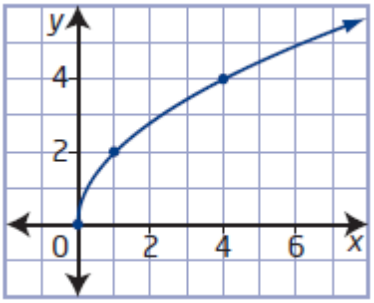
Domain and Range

Determine the domain and range of the base function, then apply the parameters to determine the domain and range of the transformed function.

$D:\left\{x \mid x \geq \frac{1}{b}x+h, x \in R\right\}$ $R:\{y \mid y \geq ay+k, y \in R\}$ ***If *a* or *b* is negative, then switch the inequality.**

Ex) Determine the domain and range of $y=-2\sqrt{3(x-5)}+4$.

Determine a Radical Function from a Graph



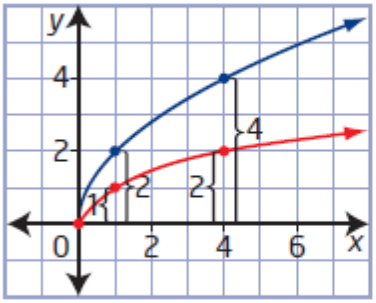
Visual approach: The stretch can be viewed as either a vertical or horizontal. Both render a correct equation.

Algebraic approach: Substitute one known coordinate of the new function into the base function and solve for either a or b, as desired.

Using the visual approach:

View as a Vertical Stretch ($y = a\sqrt{x}$)

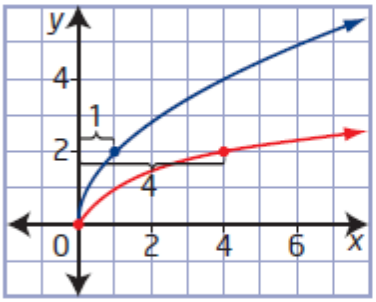
Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Using the algebraic approach:

View as a Vertical Stretch

Substitute 1 for x and 2 for y in the equation $y = a\sqrt{x}$. Then, solve for a.

$y = a\sqrt{x}$
$2 = a\sqrt{1}$
$2 = a(1)$
$2 = a$

The equation of the function is $y = 2\sqrt{x}$.

View as a Horizontal Stretch

Substitute the coordinates (1, 2) in the equation $y = \sqrt{bx}$ and solve for b.

$y = \sqrt{bx}$
$2 = \sqrt{b(1)}$
$2 = \sqrt{b}$
$2^2 = (\sqrt{b})^2$
$4 = b$

The equation can also be expressed as $y = \sqrt{4x}$.

Four equations can be developed in order to show how the function could transform into the 4 quadrants. This is achieved by reflections in the *y*-axis, as well as in the *x*-axis, then another reflection in the *y*-axis.

$y = 2\sqrt{x}$
$y = -2\sqrt{x}$
$y = 2\sqrt{-x}$
$y = -2\sqrt{-x}$

$y = \sqrt{4x}$
$y = \sqrt{-4x}$
$y = -\sqrt{4x}$
$y = -\sqrt{-4x}$

Assignment Pg 72 #2 – 4, 5abcd, 6ab, 10bc

2.1 Examples

1. Explain how to transform the graph of $y=\sqrt{x}$ to obtain $y=-2\sqrt{4(x-3)}+1$. Sketch the graph of the transformed function. Then identify the **domain** and **range** of the base function, $y=\sqrt{x}$ and of $y=-2\sqrt{4(x-3)}+1$.

$a=$ $b=$ $h=$ $k=$

The transformations applied to $y=\sqrt{x}$ to obtain $y=-2\sqrt{4(x-3)}+1$ are:

- Vertical stretch by a factor of _____
- Reflection in the _____
- Horizontal stretch by a factor of _____
- Horizontal translation _____ units _____ and vertical translation _____ units _____.

Mapping

$(x,y)\rightarrow\left(\frac{1}{b}x+h, ay+k\right)$

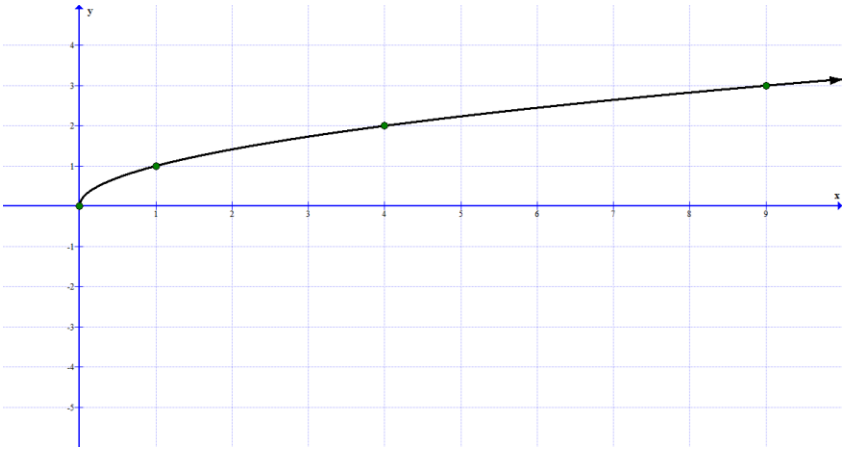
$(x,y)\rightarrow(\quad,\quad)$

$(\quad,\quad)\rightarrow$

$(\quad,\quad)\rightarrow$

$(\quad,\quad)\rightarrow$

$(\quad,\quad)\rightarrow$



Base function $y=\sqrt{x}$

Domain

Range

Transformed Function $y=-2\sqrt{4(x-3)}+1$

Domain

Range

Determine the domain and range of the base function, then apply the parameters to determine the domain and range of the transformed function.

$D:\left\{x \mid x\geq\frac{1}{b}x+h, x\in R\right\}$ $\quad R:\{y \mid y\geq ay+k, y\in R\}$

***If *a* or *b* is negative, then switch the inequality.**

2. Use **mapping** to sketch the graph of $y = 4\sqrt{-2(x+3)} - 5$.

$a =$ $b =$ $h =$ $k =$

Mapping

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

$(x, y) \rightarrow (\quad , \quad)$

$(\quad , \quad) \rightarrow$

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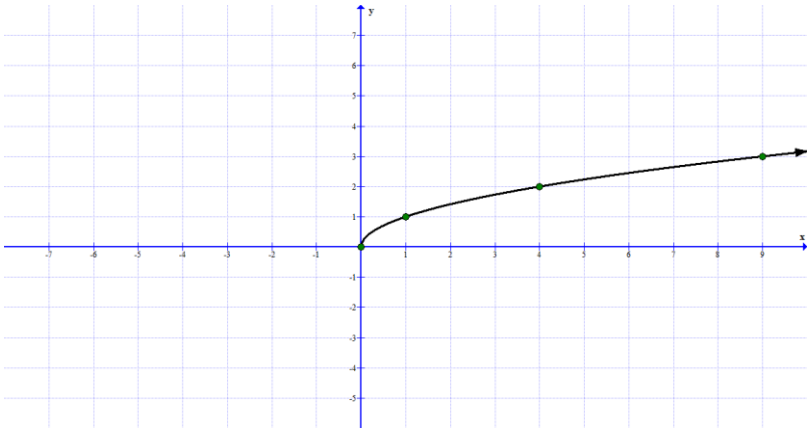


Base function $y = \sqrt{x}$

Domain

Range

Transformed Function $y = 4\sqrt{-2(x+3)} - 5$

Domain

Range

2.3 Solving Radical Equations

Example 1) Determine the **roots(s)** of $\sqrt{x+5}-3=0$.

Identify any restrictions on the variable in the radicand.

Solve the equation algebraically.

Does your response exist in the domain you identified?

Any roots that are discovered algebraically that are **not in the domain are called **extraneous roots.**

What does this situation look like? Create the graph of $y=\sqrt{x+5}$, and translate that graph down 3 units. This will give us the graph of $f(x)=\sqrt{x+5}-3$.

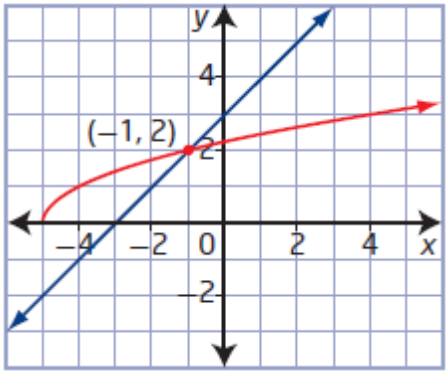
Note where the *x*-intercept is.

Your Turn! Algebraically determine the root(s) of the equation $\sqrt{x+2}-4=0$.

Example 2) Solve the equation $\sqrt{x+5} = x+3$ algebraically.

You MUST check to see if you've 'created' an extraneous root.

To solve this equation graphically, enter each side of the equation separately into a graphing program or calculator. For example, one equation is $y = \sqrt{x+5}$ (the left-hand side) and the other equation is $y = x+3$, the right-hand side. We are looking for the point where these two functions **intersect.**



The two functions intersect at the point (-1, 2). The value of x at this point is the solution to the equation.

Therefore, x = -1

Note that we write the final answer ONLY as the *x*-value, NOT the ordered pair!

Assignment Pg 96 #2a, 6abd (no restr.), 9 (no graph)

Review for the Chapter 1 and 2 exam Pg 99 #2 – 5, 7 – 10ab, 12, 16 (algebraic only)