10.1 and 10.2 Function Operations

- We will take two or three functions and combine them through addition, subtraction, multiplication and division.

Sum of Functions

$h(x) = f(x) + g(x)$ This can also be written as $h(x) = (f + g)(x)$.

Difference of Functions

$h(x) = f(x) - g(x)$ This can also be written as $h(x) = (f - g)(x)$

Examples:

1. Given the functions $f(x) = 2x + 1$ and $g(x) = x^2$, determine the function $h(x) = (f + g)(x)$. What type of function is $h(x)$? As well, find $h(3)$.

2. Given the functions $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$, determine $h(x) = f(x) - g(x)$.

3. If $h(x) = (f + g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$ when $h(x) = \sqrt{x + 7} + 5x + 2$.

Product of Functions

$h(x) = f(x)g(x)$ This can also be written as $h(x) = (f \bullet g)(x)$

Quotient of Functions

$h(x) = \dfrac{f(x)}{g(x)}$ This can also be written as $h(x) = \left(\dfrac{f}{g}\right)(x)$, **where** $g(x) \neq 0$

Examples: Given $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$, determine the following and state any non-permissible values.

a) $h(x) = (f \bullet g)(x)$

b) $h(x) = \left(\dfrac{g}{f}\right)(x)$

Assignment Pg 483 "ac" of #1 - 4, and of 9 – 11 and Pg 496 "ac" of #1, 6 – 8

10.3 Composite Functions (Day 1)

- Now we will substitute one function, for example $f(x)$ into another function, for example $g(x)$. The result of this would be $g(f(x))$. This result is read as "*g* of *f* of *x*".
- The notation for this function composition is $(g \circ f)(x)$, which **should not be confused with multiplication** which is shown as $(g \bullet f)(x)$.

Evaluate for $f(x) = 4x,\ g(x) = x + 6,\ h(x) = x^2$.

1. $f(g(3))$ There are two ways to do this:

Option 1

Determine $g(3)$.

Now subst. $g(3)$ into $f(x)$.

Option 2

Determine $f(g(x))$.

Now subst. $x = 3$ into $f(g(x))$.

2. $g(h(-2))$

3. $h(h(2))$

If $f(x) = |x|$ and $g(x) = x + 1$, determine $f(g(-11))$.

Let $f(x) = x + 1$ and $g(x) = x^2$. Determine the equation of each composite function. Sketch the graph and state the domain and range.

a) $y = f(g(x))$

b) $y = g(f(x))$

c) $y = f(f(x))$

d) $y = g(g(x))$

Assignment Pg 507 #1, 3 – 5

10.3 Composite Functions (Day 2)

Composite function and restrictions. Consider $f(x)=\sqrt{x-1}$ and $g(x)=x^2$.

a) Determine $(f\circ g)(x)$.

b) Determine $(g\circ f)(x)$.

- Does order matter when composing functions?

c) State the **domain** of:

i) $f(x)$

ii) $g(x)$

iii) $(f\circ g)(x)$

iv) $(g\circ f)(x)$

Determining the original functions from a composition.

If $h(x)=f(g(x))$, determine $f(x)$ and $g(x)$.

a) $h(x)=(x-2)^2+(x-2)+1$

Look for common elements. What do you see?

b) $h(x)=\sqrt{x^3+1}$

Start with the function inside the radical.

If $h(x)=f(g(x))$, determine $f(x)$ and $g(x)$. Here, $h(x)=\sqrt[3]{x}+\dfrac{3}{3+\sqrt[3]{x}}$.

Applications

A spherical weather balloon has a radius, r, in feet, after t minutes is given by $r = \sqrt{t}$.

a) Express the **volume** of the balloon as a function of time, t.

- Need to know that volume of a sphere is $V(r) = \frac{4}{3}\pi r^3$. Compose the new function.

b) After how many minutes will the volume be 4000 ft^3?

Same base question as above, but this time express the **surface area** of the balloon as a function of time, t.

- Recall: For a sphere: $Surface\ Area = 4\pi r^2$

After how many minutes will the surface area be 180 ft^2?

Assignment Pg 507 #6 – 8, 11a, 12, 14, 16 (1 method only), 17, 18