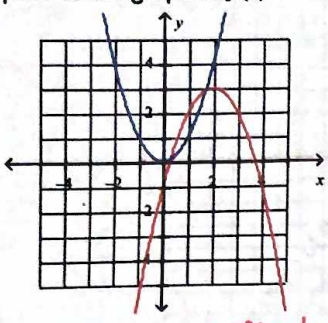
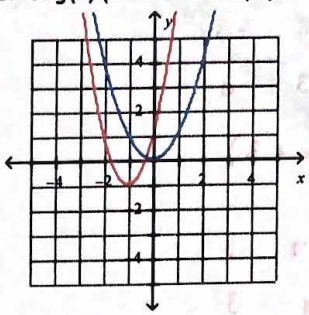
For each of the following, determine the parameters and describe the combination of transformations that must be applied to the graph of $f(x) = x^2$ (shown in blue) to obtain the graph of $g(x)$ (shown in red). (2 marks each)

a)



$a = -1 \rightarrow$ reflected over x
$h = 2 \rightarrow$ right 2
$k = 3 \rightarrow$ up 3

b)



$a = 2$
$h = -1 \rightarrow$ left 1
$k = -1 \rightarrow$ down 1

4

2. The base function $f(x) = \sqrt{x}$ is reflected in the x-axis, stretched horizontally by a factor of 2, compressed vertically by a factor of -3, and translated 3 units to the left and 5 units down.

a) Write the equation of the transformed function $g(x)$. (2 marks)

$y = -3\sqrt{\frac{1}{2}(x+3)} - 5$

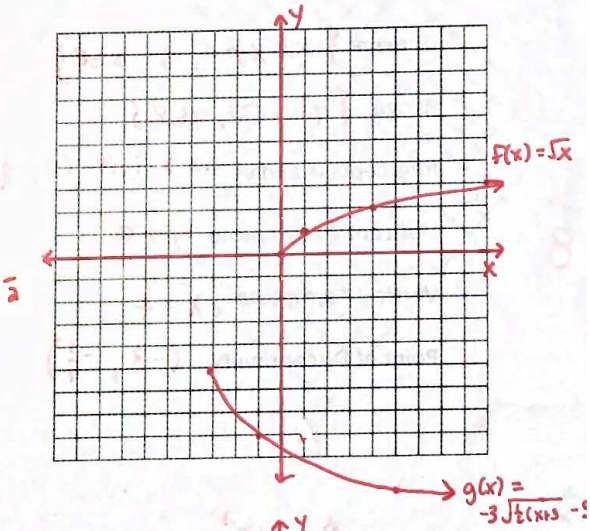
2

b) Graph the original function and the transformed function on the same set of axes. (2 marks)
Show mapping of at least 4 points. (2 marks)

$(x, y) \rightarrow (2x - 3, -3y - 5)$

$(16, 4) \rightarrow (29, -17)$
$(9, 3) \rightarrow (15, -14)$
$(4, 2) \rightarrow (5, -11)$
$(1, 1) \rightarrow (-1, -8)$
$(0, 0) \rightarrow (-3, -5)$

4



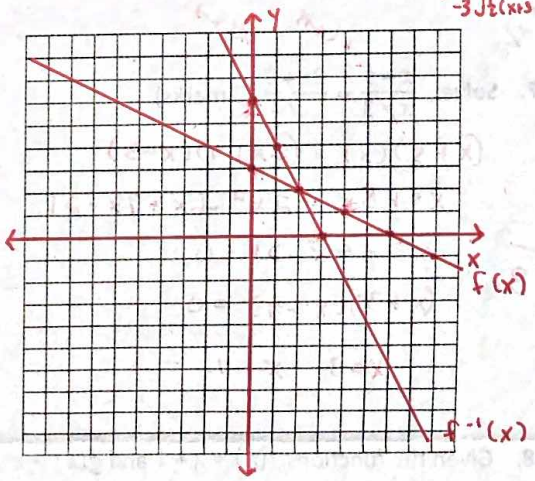
2

3. For the function $f(x) = -\frac{1}{2}x + 3$

i) determine $f^{-1}(x)$ (1 mark)

$x = -\frac{1}{2}y + 3$
$-2(x-3) = (-\frac{1}{2}y)\ -2$
$y = -2x + 6$

ii) graph $f(x)$ and its inverse (2 marks)



2

4. The height of a square-based box is 4 cm more than the side length of its square base. If the volume of the box is 225 cm³, what are its dimensions? (4 marks)

Let x = width
$(x)(x)(x+4) = 225$
$x^2(x+4) = 225$
$x^3 + 4x^2 - 225 = 0$

$x = 5$ ✓
$\therefore x - 5$

	1	4	0	-225
-5		-5	-45	-225
x	1	9	45	0

$(x-5)(\underbrace{x^2 + 9x + 45}_{\text{no factors}})$

$\therefore x = 5$

Dimensions are 5 x 5 x 9

4

17

5. Factor $x^4 + 4x^3 + 2x^2 - 4x - 3$ fully. Draw a graph of the function, including reference points. (7 marks)

$x = 1 \quad \therefore \quad x - 1$

	1	4	2	-4	-3
-1		-1	-5	-7	-3
x	1	5	7	3	0

$(x-1)(x^3 + 5x^2 + 7x + 3)$

$x = -1 \quad \therefore \quad x + 1$

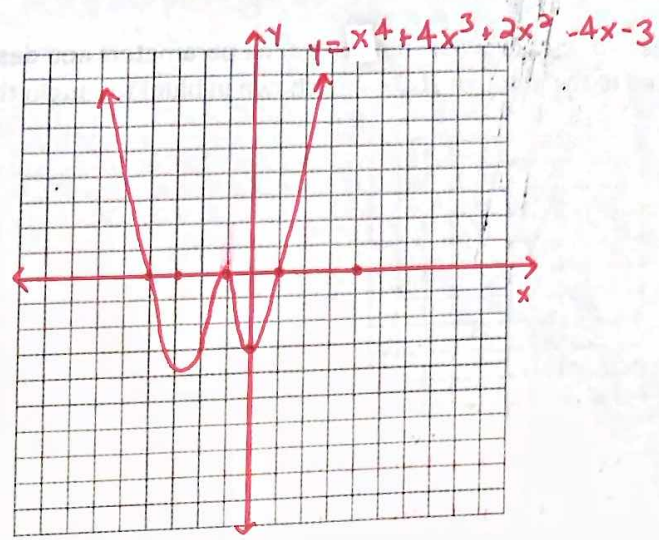
	1	5	7	3
1		1	4	3
x	1	4	3	0

$(x-1)(x+1)(x^2 + 4x + 3)$

$(x-1)(x+1)(x+3)(x+1)$

$\boxed{(x+1)^2(x-1)(x+3)}$

7



2

$-2 = -3 \quad (-2, -3)$

5

6. Consider the function $f(x) = \dfrac{x+3}{x^2 - x - 12}$. (8 marks)

a) Determine the key features of the function:

Domain $\{x \mid x \neq 4, -3, x \in R\}$

Range $\{y \mid y \neq 0, y \in R\}$

Intercepts (x and y) no x int, $(0, -\frac{1}{4})$

6

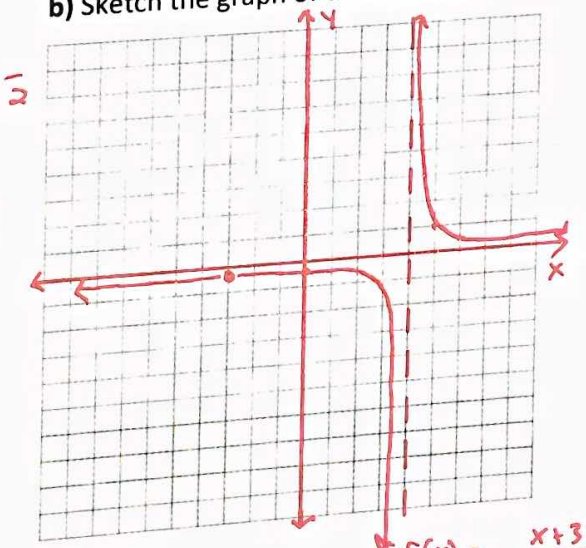
Horizontal Asymptote $y = 0$

Vertical Asymptote $x = 4$

Point of Discontinuity $(-3, -\frac{1}{7})$

8

b) Sketch the graph of the function.



2

$f(x) = \dfrac{x+3}{x^2 - x - 12}$

$\dfrac{\cancel{x+3}}{(x-4)\cancel{(x+3)}}$

$\dfrac{1}{(x-4)}$

$\dfrac{1}{6-4} = \dfrac{1}{2} \quad (6, \frac{1}{2})$

$\dfrac{1}{5-4} = \dfrac{1}{1} \quad (5, 1)$

7. Sketch the function $f(x) = \frac{x-2}{x^2-6x+8}$. Identify the following characteristics: (9 marks)

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 0$

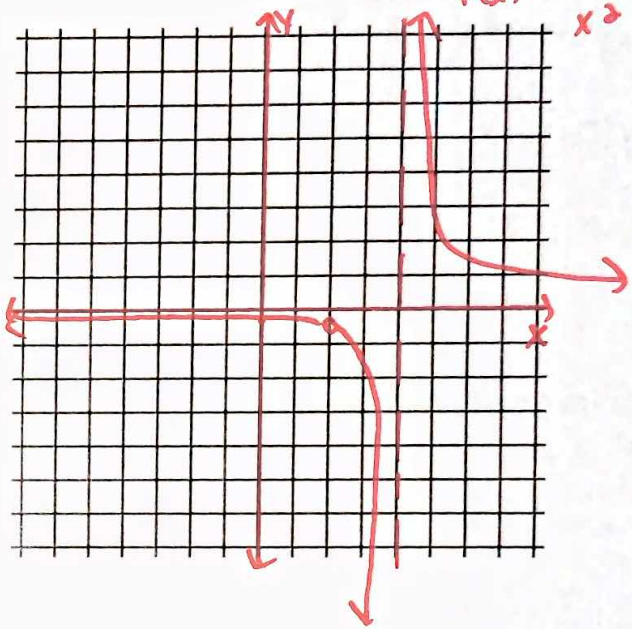
Point(s) of discontinuity: $(2, -\frac{1}{2})$

$\frac{1}{2-4} = -\frac{1}{2}$

x-intercept(s): None

y-intercept(s): $(0, -\frac{1}{4})$

$f(x) = \frac{x-2}{x^2-6x+8}$



$\frac{\cancel{x-2}}{(\cancel{x-2})(x-4)} = \frac{1}{x-4}$

8. Solve: a) $\frac{x+5}{\cancel{x-3}} = \frac{2x+7}{\cancel{x}}$ $\cdot x(\cancel{x-3})$ $\cdot x(x-3)$

$x^2 + 5x = 2x^2 - 6x + 7x - 21$

$x^2 - 4x - 21 = 0$

$(x+3)(x-7) = 0$

$\boxed{x = -3 \quad x = 7, x \neq 0, 3}$

b) $-2 = 3\sqrt{x+4} - 5$

$\frac{3}{3} = \frac{3\sqrt{x+4}}{3}$

$(1)^2 = (\sqrt{x+4})^2$

$1 = x+4$

$\boxed{x = -3}$

$x+4 \geq 0$

$x \geq -4$

Check:

$-2 = 3\sqrt{-3+4} - 5$

$-2 = 3\sqrt{1} - 5$

$-2 = -2$ ✓