ANSWERS 1.4

1.
(a) $(0.43)(0.60) = 0.258$
(b) $\left(\frac{7}{10}\right)\left(\frac{5}{14}\right) = \frac{1}{4}$

2.
(a) $\left(\frac{1}{2}\right)\left(\frac{1}{6}\right) = \frac{1}{12}$
(b) $\left(\frac{1}{2}\right)\left(\frac{3}{6}\right) = \frac{1}{4}$
(c) $\left(\frac{1}{2}\right)\left(\frac{2}{6}\right) = \frac{1}{6}$
(d) $\left(\frac{1}{2}\right)\left(\frac{0}{6}\right) = 0$

3.
(a) $\left(\frac{3}{8}\right)\left(\frac{2}{8}\right) = \frac{3}{32}$
(b) $\left(\frac{5}{8}\right)\left(\frac{6}{8}\right) = \frac{15}{32}$
(c) $\left(\frac{5}{8}\right)\left(\frac{8}{8}\right) = \frac{5}{8}$
(d) $\left(\frac{5}{8}\right)\left(\frac{0}{8}\right) = 0$

4.
(a) $\left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = \frac{9}{256}$
(b) $\left(\frac{6}{16}\right)\left(\frac{6}{16}\right) = \frac{9}{64}$
(c) $P[(R \cap R) \cup (Y \cap Y) \cup (G \cap G) \cup (B \cap B)]$
$= \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) + \left(\frac{5}{16}\right)\left(\frac{5}{16}\right) + \left(\frac{2}{16}\right)\left(\frac{2}{16}\right) + \left(\frac{6}{16}\right)\left(\frac{6}{16}\right)$

$= \frac{9+25+4+36}{256} = \frac{37}{128}$
(d) $1 - \frac{37}{128} = \frac{91}{128}$
(e) $P[(R \cap Y) \cup (Y \cap R)]$
$= \left(\frac{3}{16}\right)\left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)\left(\frac{3}{16}\right) = \frac{15}{128}$
(f) $P[(B \cap G) \cup (G \cap B)]$
$= \left(\frac{6}{16}\right)\left(\frac{2}{16}\right) + \left(\frac{2}{16}\right)\left(\frac{6}{16}\right) = \frac{3}{32}$

5.
(a) $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{1296}$
(b) $\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) = \frac{625}{1296}$
(c) $1 - \frac{625}{1296} = \frac{671}{1296}$
(d) $\left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = \frac{1}{16}$
(e) Answer: $\frac{3}{8}$. Let event D be the rolling of an odd number and let event E be the rolling of an even number. Then the probability of rolling two odds followed by two evens is
$P(D \cap D \cap E \cap E) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = \frac{1}{16}$.
But the events DDEE can be arranged in $\frac{4!}{2! \cdot 2!}$ or 6 ways. Thus the probability of two odds and two evens in any order is $6\left(\frac{1}{16}\right) = \frac{3}{8}$.

6. Answer: 26 rolls. Let n be the number of rolls. Then the probability of not rolling a 4 in n rolls is $\left(\frac{5}{6}\right)^n$. Thus the probability of rolling at least one 4 in n rolls is $1 - \left(\frac{5}{6}\right)^n$. We require $1 - \left(\frac{5}{6}\right)^n \geq 0.99$. Rearranging we have $\left(\frac{5}{6}\right)^n \leq 0.01$. By trial and error, we find that n is 26.

7.
(a) $(0.95)(0.92) = 0.874$
(b) $P(M \cup C) = P(M) + P(C) - P(M \cap C)$
$= 0.95 + 0.92 - 0.874 = 0.996$

8.
(a) $\left(\frac{1}{36}\right)\left(\frac{1}{36}\right) = \frac{1}{1296}$
(b) $\left(\frac{5}{36}\right)\left(\frac{5}{36}\right) = \frac{25}{1296}$

ANSWERS 1.4 (cont'd)

9.

(a) Answer 0.98425. The probability that you do not finish first in any of the three sports is $(0.30)(0.35)(0.15) = 0.01575$. Then the probability that you do finish first in at least one of the three sports is $1 - 0.01575 = 0.98425$.

(b) $P(F \cap B \cap \overline{S}) = (0.70)(0.85)(0.35) = 0.20825$

(c) $P(F \cap S) = P(F) \cdot P(S) = (0.70)(0.65) = 0.455$

(d) Answer: 0.15525.

There are three cases, each of whose probabilities is worked out below. Since any of these cases is possible, the probability that the school finishes first in exactly one of the three sports is the sum of these probabilities.

$P(F \cap \overline{S} \cap \overline{B}) = P(F) \cdot P(\overline{S}) \cdot P(\overline{B})$
$= (0.70)(0.35)(0.15) = 0.03675$

$P(\overline{F} \cap S \cap \overline{B}) = P(\overline{F}) \cdot P(S) \cdot P(\overline{B})$
$= (0.30)(0.65)(0.15) = 0.02925$

$P(\overline{F} \cap \overline{S} \cap B) = P(\overline{F}) \cdot P(\overline{S}) \cdot P(B)$
$= (0.30)(0.35)(0.85) = 0.08925$

10.

(a) $(0.85)^{20} = 0.03876$

(b) 0.96124

11. $(0.98724)^{14}(0.99586)^{16} = 0.78179$

12.

(a) $P(W \cap W \cap W \cap W) = [P(W)]^4$
$= (0.6)^4 = 0.12960$

(b) Answer: 0.20736. The last game must be a win by the Blades. One possible order is $W \cap W \cap L \cap W \cap W$.
$P(W \cap W \cap L \cap W \cap W) = [P(W)]^4[P(L)]$
$= (0.6)^4(0.4) = 0.05184$. The 3 wins and 1 loss in the first 4 games can take place in $\frac{4!}{3!}$ or 4 orders. Thus the probability that the Blades win the series in exactly 5 games is $4(0.05184) = .20736$.

(c) Answer: 0.20736. The last game must be a win by the Blades. One possible order is

12 (c) (cont'd)

$W \cap W \cap L \cap W \cap L \cap W$.
$P(W \cap W \cap L \cap W \cap L \cap W) = [P(W)]^4[P(L)]^2$
$= (0.6)^4(0.4)^2 = 0.020736$. The 3 wins and 2 losses in the first 5 games can take place in $\frac{5!}{3! \cdot 2!}$ or 10 orders. Thus the probability that the Blades win the series in exactly 6 games is $10(0.020736) = 0.20736$.

(d) Answer: 0.16589. The last game must be a win by the Blades. One possible order is $L \cap W \cap L \cap W \cap L \cap W \cap W$.
$P(L \cap W \cap L \cap W \cap L \cap W \cap W) = [P(W)]^4[P(L)]^3$
$= (0.6)^4(0.4)^3 = 0.0082944$. The 3 wins and 3 losses in the first 6 games can take place in $\frac{6!}{3! \cdot 3!}$ or 20 orders. Thus the probability that the Blades win the series in exactly 7 games is $20(0.0082944) = 0.16589$.

(e) Answer: 0.71021. The probability that the Blades will win the series in 4, 5, 6, or 7 games is the sum of the individual probabilities in parts (a) to (d) since these are mutually exclusive events.

13. Answer 0.51086. The probability to win if you place once is $\frac{{}_6C_6}{{}_{49}C_6} = \frac{1}{13983816}$. The probability that you do not win, if you play once, is $\frac{13983815}{13983816}$. The probability you do not win if you place 10 000 000 times is $\left(\frac{13983815}{13983816}\right)^{10000000}$ or 0.48914. Thus the probability that you do win at least once is $1 - 0.48914$ or 0.51086.

14. to be discussed

15. If events A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$. If A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$.