Pre-Calculus 20 Chapter 7 Notes

Section 7.1 Absolute Value

Absolute value is how far a number is from zero. Distances are always positive values.

How far is 4 from zero? ___________

How far is -4 from zero? ___________

The symbol for absolute value is | |

$|4| = 4$

$|-4| = 4$

Formal Definition of Absolute Value:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \quad \text{(leave x as is if it is positive)} \\ -x, & \text{if } x < 0 \quad \text{(multiply by -1 if it is negative)} \end{cases}$$

Ex1: Evaluate each of the following. Watch your order of operations.

Absolute value is done in the same order as brackets in BEDMAS.

$|5|$ $\quad$ $|-7|$ $\quad$ $|2 - 4|$ $\quad$ $|5(-3.5)|$ $\quad$ $3 + |-2|$

Ex2: Evaluate each of the following. Watch your order of operations.

Absolute value is done in the same order as brackets in BEDMAS. A number placed in front of the absolute value mean multiply (just like a number in front of brackets).

$|-4| + |-3|$ $\quad$ $-2|-3|$ $\quad$ $2 - 3|-12 + 8|$ $\quad$ $-2|-2(5 - 7)^2 + 6|$

When trying to find the distance between 2 numbers, we use the absolute value to ensure that the answer will be a positive result since distance is always positive.

RECALL: We subtract 2 numbers to find the distance between them.

Ex3: The highest temperature this month was 19°C. The coldest temperature was a whopping -30°C. What is the total temperature difference?

$|19 - (-30)| = |19 + 30| = |49| = 49$

OR

$|-30 - 19| = |-49| = 49$ Notice that by using Absolute value, the order we subtract doesn't affect the final answer.

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7.2 Graphing the Absolute Value Function

Learning what the basic absolute value function looks like!

Graph $y = x$.

Table of Values

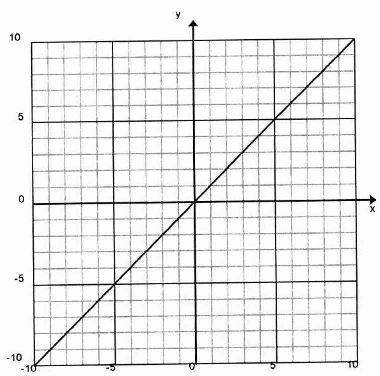
x	y
3	3
2	2
1	1
0	0
-1	-1
-2	-2
-3	-3

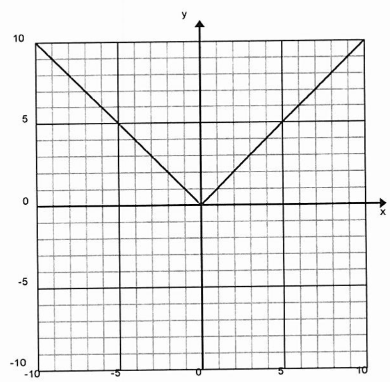
Graph $y = |x|$

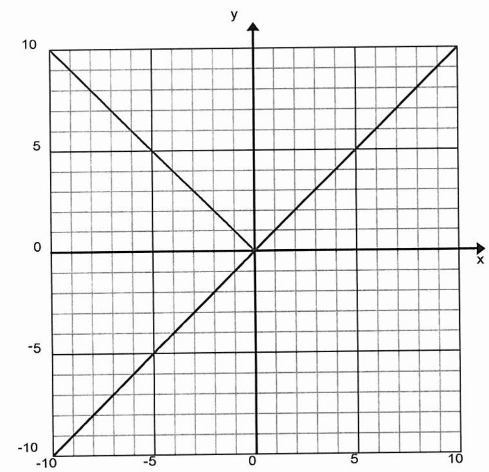
Table of Values

x	y
3	3
2	2
1	1
0	0
-1	1
-2	2
-3	3

$y = x$ and $y = |x|$ graphed together.







Notice that for the Absolute Value table, all the y values are positive. (definition of absolute value)

Describe the similarities and differences between $y = x$ and $y = |x|$.

The part of the graph where $x \geq 0$ is identical for both graphs.
The part of the graph where $x < 0$ has differences:

- in $y = x$, the y values are negative
- in $y = |x|$, the y values are positive (reflection across the x-axis)

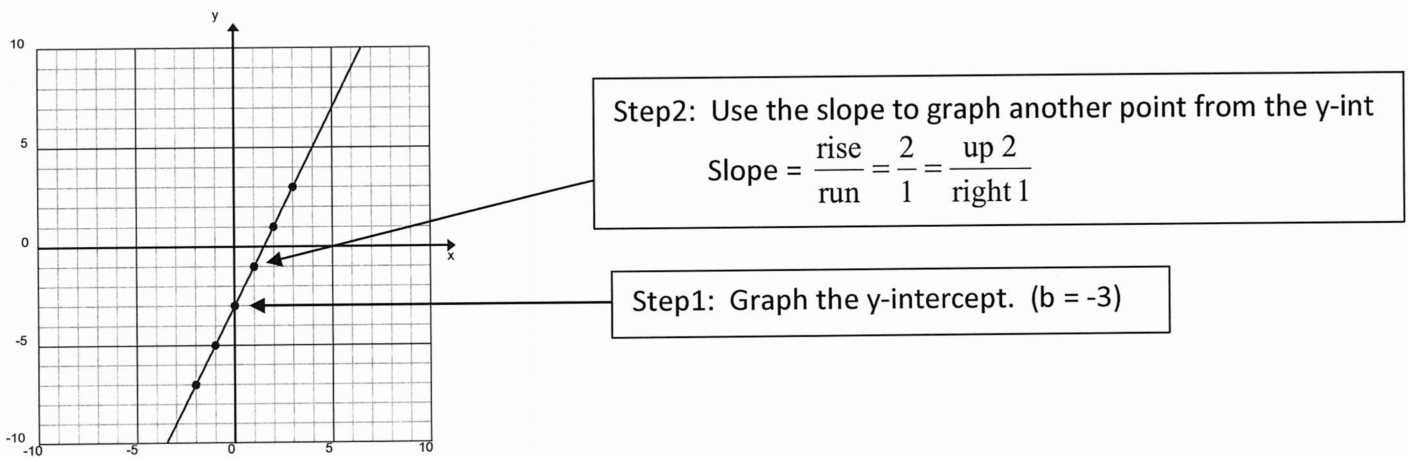
Formal Definition of Absolute Value Function:

$$y = \begin{cases} x, & \text{if } x \geq 0 & \text{(leave x as is if it is positive)} \\ -x, & \text{if } x < 0 & \text{(multiply by -1 if it is negative)} \end{cases}$$

Part 1: Graphing Absolute Values Functions of the form $y = |ax + b|$ {**Linear Functions**}

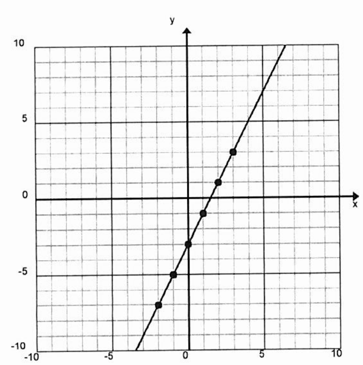
Recall how to **Graph Linear Functions** written in the form $y = ax + b$

Step 1: Graph the y-intercept point (b value)
Step 2: Use the slope to graph another point.
Step 3: Continue using the slope to graph more points in both directions (pos and neg)



Example 1: Graph $y = |2x - 3|$

Step 1: Graph the function $y = 2x - 3$ (ignore the absolute value for now)
previous example. (**Graph at least 3 negative "y" values.**)



Step 2: Calculate the exact value of the x-intercept (where the line crosses the x-axis)
Recall that to do this, we set y = 0 and then solve for x.

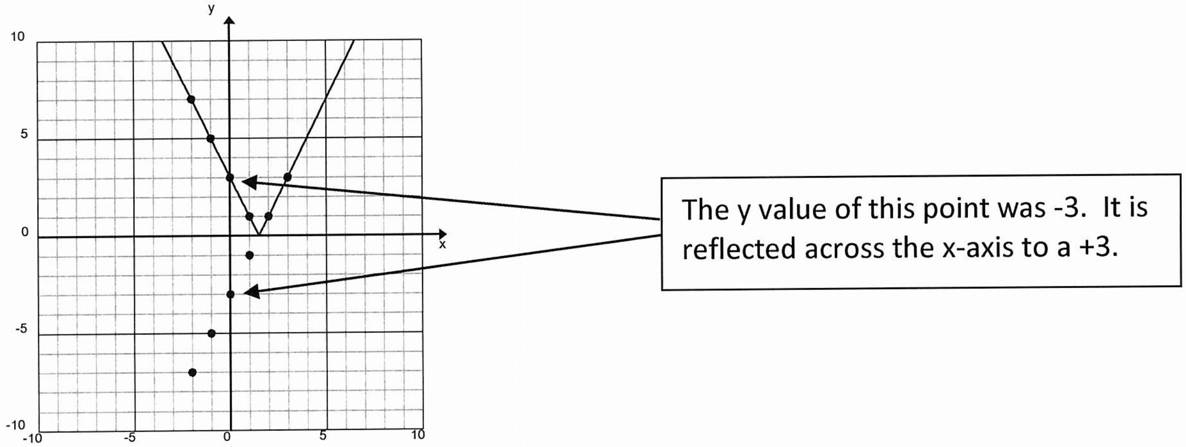
$y = 2x - 3$

$0 = 2x - 3$

$-2x = -3$

$x = \frac{3}{2}$ so the x-intercept is (1.5, 0)

Step 3: Reflect the y-values that are negative across the x-axis. The resulting V-Shaped graph is the graph of y = |2x – 3|



Step 4: Write out the Domain and Range of the graph y = |2x – 3|

D: {**x | x∈R**}
R: {**y | y∈R | y≥0** }

Example 2: Write out the function y= |2x – 3| as a piecewise function.

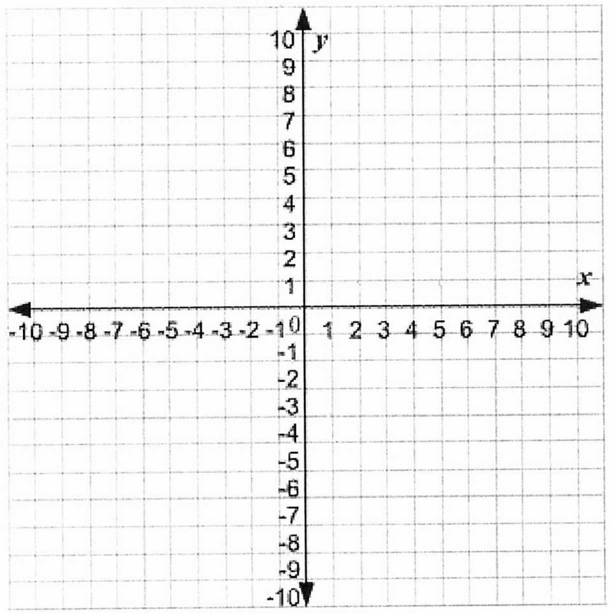
A piecewise function is ____.

$$y = \begin{cases} \mathbf{2x - 3\text{, if } x \geq 1.5} & \text{(leave function as is when the graph is above the x-axis)} \\ \mathbf{-(2x - 3)\text{, if } x < 1.5} & \text{(multiply the function by -1 when the graph is below the x-axis)} \end{cases}$$

or -2x + 3 is correct also for the second part {you can distribute the negative}

Example 3

Graph $y = \left|-\frac{1}{2}x + 1\right|$



Write $y = \left|-\frac{1}{2}x + 1\right|$ as a piecewise function.

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7.2 Graphing the Absolute Value Function (continued)

Graphing Absolute Values of Quadratic Functions of the form $y = |ax^2 + bx + c|$

Step 1: Find the x-intercepts.
Step 2: Find the vertex. (Recall: Vertex is always at the midpoint of the x-intercepts) Step3: Graph the vertex and x-intercepts and find at least 2 more values to graph.
(Use Staircase Method or Table of Values) Example:

Graph $y = |x^2 - x - 2|$

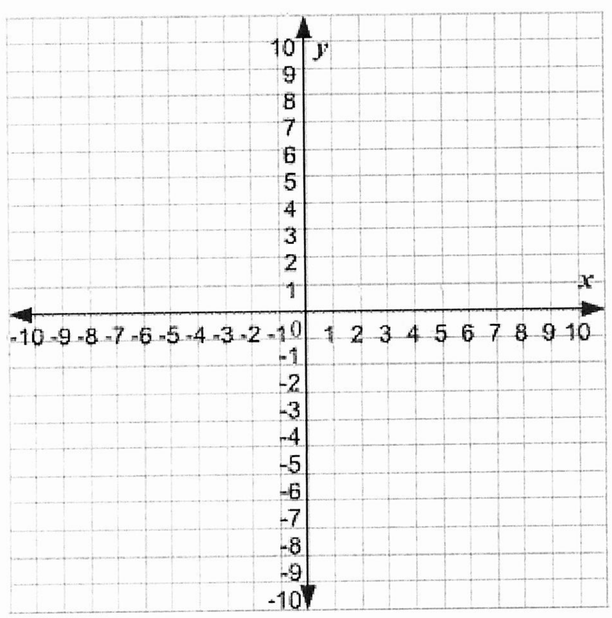
Ignore the absolute value for now, and graph as we did previously.

Step 1: To find the x-intercepts, set y = 0 and solve for x. If possible, solve by factoring. If it is not factorable, then you must use the quadratic formula.

Step 2: Find the vertex.

Step 3: Graph the x-intercepts and vertex. Then find more values to create a more accurate graph of the function. (Use Staircase Method 1a, 3a, 5a or Table of Values)

Step 4: Reflect the y-values that are negative across the x-axis. The resulting graph is the graph of $y = |x^2 - x - 2|$. You can choose to erase the part of the graph that is below the x-axis.

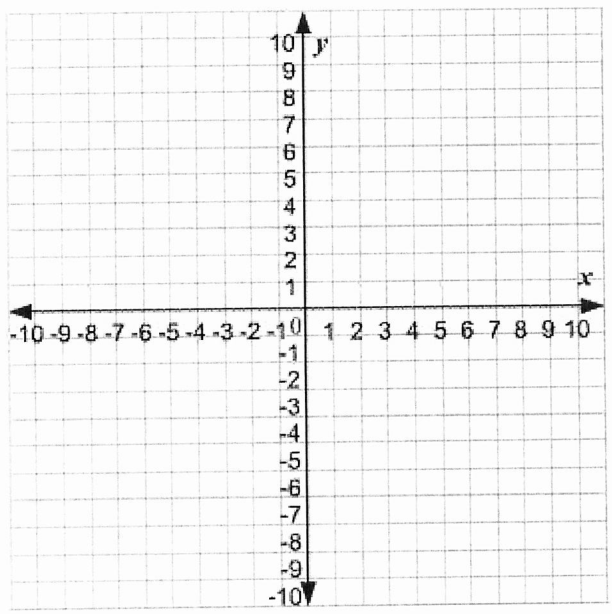


Domain: {**x | x∈R**}
Range: {**y | y∈R | y≥0** }

Write out the function $y = | x^2 - x - 2 |$ as a piecewise function.

Extra Practice:

Graph the function $y = |-2x^2 - 3x + 9|$ and write the piecewise function.



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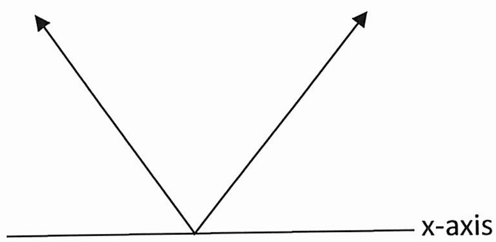
Extra Notes for Section 7.2

Key Points to Remember:

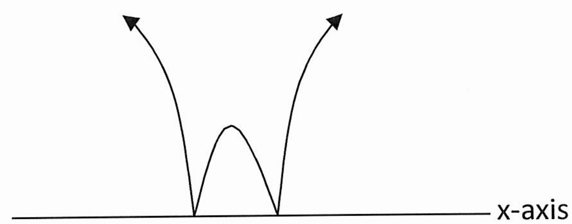
$y = |\text{linear}|$

General look of graph:



$y = |\text{quadratic}|$

General look of graph:



Write the **piecewise function** for each of the following:
(Keep in mind the general look of the graph when deciding the "if" sections of the piecewise functions.) Remember to **find the x-intercepts** first!

1. $y = |2x - 1|$

x-int: $0 = 2x - 1$

$-2x = -1$

$x = \frac{1}{2}$

$$y = \begin{cases} \mathbf{2x - 1}, & \textbf{if } x \geq \frac{1}{2} \quad \textbf{(pos sloped part)} \\ \mathbf{-(2x - 1)}, & \textbf{if } x < \frac{1}{2} \quad \textbf{(neg sloped part)} \\ \mathbf{-2x + 1} & \qquad\qquad \textbf{was reflected} \end{cases}$$

2. $y = |-\frac{1}{2}x + 2|$

$$y = \begin{cases} & \\ & \end{cases}$$

3. $y = |x^2 - 16|$

x-int: $0 = x^2 - 16$

$0 = (x + 4)(x - 4)$

$x = -4$ and 4

$$y = \begin{cases} \mathbf{x^2 - 16}, & \textbf{if } x \leq 4 \textbf{ and } x \geq 4 \quad \textbf{(outer parts)} \\ \mathbf{-(x^2 - 16)}, & \textbf{if } -4 < x < 4 \quad \textbf{(inner part)} \\ & \qquad\qquad \textbf{was reflected} \end{cases}$$

4. $y = |-x^2 + 1|$

$$y = \begin{cases} & \\ & \end{cases}$$

7.3 Absolute Value Equations

Recall the formal definition of Absolute Value:

How to Solve an Absolute Value Equation Algebraically:

Isolate the absolute value

Set up 2 cases

Positive Case

Negative Case

Check for extraneous solutions

Example 1:

Solve $|x-2|+3=9$

Example 2:

Solve $|x-3|=-4$

Example 3:

Solve $|x^2 - 7x + 2| = 10$

How to Solve an Absolute Value Equation Graphically:

Graph the left side of the equation.

Graph the right side of the equation.

Where the graphs intersect (x-coordinate) is the solution.

Example 4:

Solve graphically: $|2x - 5| = 5 - 3x$

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