6.1 Rational Expressions

Rational expression – an algebraic expression with a numerator and a denominator that are polynomials.

Example:

Non-permissible values – a value for a variable that makes the expression undefined. In rationals, the denominator ≠ 0.

Example: What are the non-permissible values for x in the following?

1. $\frac{6-x}{2x}$

2. $\frac{3}{x-7}$

3. $\frac{4x-1}{x^2+4x+3}$

4. $\frac{5t}{4sr^2}$

Simplifying Rational Expressions

- divide out common factors in the numerator and denominator.

Example: Simplify and state non-permissible values.

1. $\frac{9}{12}$

2. $\frac{12m^2t^5}{3mt}$

3. $\dfrac{3x-6}{x-2}$

4. $\dfrac{x^2+2x-15}{x-3}$

5. $\dfrac{2y^2+y-10}{y^2+3y-10}$

6. $\dfrac{x^2-10x+24}{x^2-6x}$

7. $\dfrac{1-t}{t^2-1}$

8. $\dfrac{25-x^2}{x^2-3x-10}$

NOTE:

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Section 6.1 Rational Expressions

1. Determine the non-permissible value(s)

a) $\dfrac{5}{x+3}$

b) $\dfrac{7}{xy}$

c) $\dfrac{x+3}{(x+4)(x-5)}$

d) $\dfrac{1-x}{3x+5}$

e) $\dfrac{2a}{a^2-3a}$

f) $\dfrac{m+1}{m^2+5m+6}$

2. Simplify each rational expression. State any non-permissible values.

a) $\dfrac{3(x+5)}{(x+5)(x-5)}$

b) $\dfrac{(x-7)(x+2)}{-5x(7-x)}$

c) $\dfrac{(x+3)^2}{3(x+3)(x-3)}$

d) $\dfrac{3x(4x-1)}{(4x-1)(3x+1)}$

e) $\dfrac{25(x-5)(x+1)}{10(2x+1)(x-5)}$

3. Simplify.

a) $\dfrac{6r^2st}{10rs^2t^2}$

b) $\dfrac{3x-6}{x^2-4}$

c) $\dfrac{x^2+x}{x^2-4x-5}$

d) $\dfrac{y^2-2y-3}{y^2-3y}$

e) $\dfrac{3x^2+11x-4}{x^2+8x+16}$

f) $\dfrac{4+8a+4a^2}{16-16a^2}$

g) $\dfrac{2x^2+5x+2}{5x^2-5x-30}$

h) $\dfrac{12x^2+4x}{3x^2-5x-2}$

i) $\dfrac{15x^3+5x^2}{6x^2-13x-5}$

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6.2 Multiplying Rationals

- Factor numerators and denominators
- Divide out common terms

Examples: Multiply. State restrictions.

1. $\frac{5}{8} \bullet \frac{2}{12}$

2. $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^5}{8}\right)$

3. $\left(\frac{a^2-a-12}{a^2-9}\right)\left(\frac{a^2-4a+3}{a^2-4a}\right)$

4. $\frac{x^2-25}{x^2-49} \bullet \frac{x^2-6x-7}{x^2+6x+5}$

Assignment: Page 327 #1, 2

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6.2 Dividing Rationals

- Multiply by the reciprocal of the rational following the division sign
- Divide out common terms only when multiplying
- Restrictions apply to anything in the denominator at any time

Reciprocals

- A rational multiplied by its reciprocal = 1
- Interchange the numerator and denominator
- Examples: $\frac{1}{2} \rightarrow$, $\frac{x^2}{y} \rightarrow$, $\frac{a+5}{a} \rightarrow$

Examples: Divide and simplify. State restrictions.

1. $\frac{6}{5} \div \frac{3}{2}$

2. $\frac{3x^2}{y^2} \div \frac{x}{y}$

3. $\frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$

4. $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \bullet \frac{2x-6}{x+4}$

Assignment: Page 327 # 3, 8, 15

6.2 Multiplying and Dividing Rationals

Do ALL work on looseleaf

1. Simplify.

a) $\left(\frac{9x}{14y^2}\right)\left(\frac{7y^3}{3x^2}\right)$

b) $\left[\frac{5xy}{(x+y)^2}\right]\left[\frac{x(x+y)}{10}\right]$

c) $(x-3)\left(\frac{x+2}{4x-12}\right)$

d) $\left[\frac{(x+1)(x-6)}{(x-6)(x+6)}\right]\left[\frac{x(x+6)}{(1+x)}\right]$

2. Write each product in simplest form.

a) $\left(\frac{x-2}{x^2-4}\right)\left(\frac{x^2-2x-8}{x+2}\right)$

b) $\left(\frac{5y-5}{y^2+4y-5}\right)\left(\frac{y^2-25}{y^2-2y-15}\right)$

c) $\left(\frac{x^3-9x}{2x^2-x-15}\right)\left(\frac{2x^2+x-10}{x^2+x-6}\right)$

d) $\left(\frac{4x^2-25}{24x^2+52x-20}\right)\left(\frac{15x^2+65x+20}{2x^2+3x-20}\right)$

3. Divide. Express each quotient in simplest form.

a) $\left(\frac{5a}{3b}\right) \div \left(\frac{15c}{9a^2}\right)$

b) $\left(\frac{x+1}{3x+5}\right) \div \left(\frac{x+3}{3x+5}\right)$

c) $\frac{4a}{3(a-4)} \div \frac{16a^2}{9(a-4)}$

d) $\frac{2(x+3)(x-3)}{3x} \div x(x-3)$

4. What are the non-permissible values for the quotient $\frac{x^2+8x+16}{(x-3)(x+5)} \div \frac{3x^2-3}{(x+4)}$. Explain your answer.

5. Simplify each quotient.

a) $\frac{16a^2b}{a^2-2a} \div \frac{4ab^2}{a^2b-4b}$

b) $\frac{x-x^2}{10x+8} \div \frac{(x-1)^2}{5x^2+4x}$

c) $\frac{x^2+8x+7}{x^2-6x-7} \div \frac{x^2+7x+6}{x^2-x-42}$

d) $\frac{9y^2-1}{y+3} \div \frac{3y^2-8y-3}{9-y^2}$

6. Simplify.

a) $\left(\frac{4x^2-4}{25x^2-50x+25}\right)\left(\frac{5x^2}{x^2+x}\right) \div (x+1)$

b) $\frac{x^2-144}{12x^2} \div \left(\frac{x^2-x-6}{x^2-2x}\right)\left(\frac{x^2+4x+4}{x^2+10x-24}\right)$

c) $\frac{9x-27}{9-x^2} \div \frac{3x+27}{x^3+3x^2} \div \frac{3x^2-3x}{x^2+6x-27}$

d) $\frac{8x^3+8x^2}{4x^2+8x+4} \div \frac{2x^2-32}{3x^2-12x} \div \frac{12x^3}{x^2+5x+4}$

7. A rectangle has an area of $2x^2 - x - 1$.

a) Determine an expression for the width of the rectangle if the length is $2x + 1$.

b) State any non-permissible value(s).

c) Are there any other non-permissible values of x? Explain.

8. a) What is the product of $\frac{x+3}{x}$ and $\frac{x+3}{x-3}$?

b) What is the quotient of $\frac{x+3}{x}$ and $\frac{x+3}{x-3}$?

c) How do the non-permissible values of the product compare with the non-permissible values of the quotient? Explain.

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6.3 Adding/Subtracting Rationals (Day 1)

- Must have a common denominator (add numerators, leave denominators)
- Answer in simplest form

Example:

1. $\frac{2}{7}+\frac{3}{7}$

2. $\frac{3}{5}+\frac{1}{10}$

3. $\frac{7x+1}{x}+\frac{5x-2}{x}$

4. $\frac{2a}{b}-\frac{a-1}{b}$

5. $\frac{2x}{x+4}+\frac{8}{x+4}$

6. $\frac{x^2}{x-2}+\frac{3x}{x-2}-\frac{10}{x-2}$

Assignment: Page 336 #1, 2, 5

Pre-Calculus 20 **6.3 Adding/Subtracting Rationals (Day 2)**

To find a common denominator, factor denominators first.

Examples:

1. $\frac{4}{2a}+\frac{5}{3a}$

2. $\frac{1}{2s}-\frac{s}{4t}+6$

3. $\frac{y^2-20}{y^2-4}+\frac{y-2}{y+2}$

4. $\frac{4}{p^2-1}+\frac{3}{p+1}$

5. $\frac{x-1}{x^2+x-6}-\frac{x-2}{x^2+4x+3}$

Assignment: Page 336 #6 a-e, 7ab, 25 b

6.3 Adding/Subtracting Rationals

Wherever applicable, state the non-permissible values for the variables.

1. State the least common denominator.

a) $\frac{9x+y}{4x}+\frac{3y}{5y}$

b) $\frac{1}{x+4}-\frac{5}{3x+1}$

c) $\frac{9}{x^2-36}+\frac{3x}{x-6}$

2. Add or subtract. Express the answers in simplest form.

a) $\frac{x+1}{3x}+\frac{4x-5}{3x}$

b) $\frac{4x^2}{x+5}+\frac{x+1}{x+5}-\frac{x^2}{x+5}$

c) $\frac{7x+6}{(x+2)(x-2)}-\frac{3x-2}{(x+2)(x-2)}$

3. Simplify.

a) $\frac{a+5}{7a}+\frac{4}{3a^2}$

b) $\frac{x-4}{5xy}-\frac{3x+1}{y^2}$

c) $\frac{x+1}{xy^2}+\frac{4}{7xy}-\frac{x-3}{5y^2}$

4. Add or subtract, and express the answers in simplest form.

a) $\frac{3}{x-5}+\frac{2}{x+7}$

b) $\frac{3x}{7y}-\frac{x}{7(y+3)}$

c) $\frac{5x}{x+1}-\frac{x^2+4}{(x+1)(x-1)}+\frac{3}{x-1}$

d) $\frac{3x}{(x+2)(x-5)}-\frac{x+3}{(x-5)(x+6)}$

5. Simplify.

a) $\frac{2a}{2a+6}-\frac{a^2+9}{a^2-9}$

b) $\frac{3y}{y^2-4}+\frac{6y}{y^2+5y+6}$

c) $\frac{3}{4-x^2}+\frac{5}{x^2+4x+4}$

d) $\frac{x-6}{x^2-11x+28}-\frac{x-5}{x^2-8x+7}$

6. Simplify each rational expression, and then add or subtract. Express the answers in simplest form.

a) $\frac{8}{3x-18}-\frac{x+1}{x^2-5x-6}$

b) $\frac{x^2-49}{x^2-8x+7}+\frac{2-2x}{x^2-1}$

c) $\frac{2x+8}{x^2+5x+6}-\frac{x-9}{(x+3)(x-5)}$

d) $\frac{2x}{x-3}-\frac{3(x+1)(x-6)}{3x^2+6x-45}+\frac{4}{x+5}$

7. Simplify.

a) $\frac{\frac{1}{x}+1}{1-\frac{1}{x}}$

b) $\frac{\frac{x}{x-3}}{4-\frac{x}{x-3}}$

c) $\frac{\frac{1}{4+h}-\frac{1}{4}}{h}$

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6.4 Rational Equations

- an equation containing at least one rational (fractional) expression
- To solve, find the LCD and multiply each term by the LCD to eliminate the fractions.
- Check. There may be extraneous roots.

Examples:

1. $\frac{x}{4}-\frac{7}{x}=3$

2. $\frac{2x}{x-4}=\frac{10}{x-4}$

3. $\frac{x}{x+3}+1=\frac{12}{x+3}$

4. $\frac{9}{y-3}-\frac{4}{y-6}=\frac{18}{y^2-9y+18}$

5. $\frac{3x}{x+2}-\frac{5}{x-3}=\frac{-25}{x^2-x-6}$

Assignment: Page 348 # 1, 2, 3

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6.4 Rational Word Problems

- Use the 5-step method (write "let" statements, write equation, solve, check, write final statement)

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#9. Let x = first number
Let x + 1 = second number

Assignment: Page 348 #8, 11

6.4 Extra Practice

1. Solve and check each equation.

a) $\frac{2x}{3} = \frac{x}{4} + \frac{5}{6}$

b) $\frac{5}{2a} = 3 - \frac{2}{a}$

c) $\frac{x-3}{x^2} = \frac{11}{15x} - \frac{2}{5x}$

d) $\frac{x^2}{3} + \frac{11x}{18} = \frac{1}{9}$

e) $2x = \frac{3}{x} - \frac{5}{2}$

f) $\frac{x}{3} = \frac{2}{x} + \frac{x+1}{3}$

2. Solve each rational equation.

a) $\frac{3}{x+3} = \frac{x+15}{x+3} - 5$

b) $\frac{x}{x+1} - \frac{x+4}{x+1} = \frac{6}{x}$

c) $\frac{2}{x-3} + \frac{3}{x} = 2$

d) $\frac{x}{x-2} + \frac{2}{x+2} = 1$

3. Solve.

a) $\frac{21}{5x+3} = -3$

b) $\frac{x+1}{x-3} = \frac{x}{x-5}$

c) $\frac{x+4}{x-2} = \frac{x-4}{x+4}$

d) $\frac{x-2}{x} = \frac{2-x}{x+1}$

4. Solve the following equations.

a) $\frac{x}{x-3} + \frac{x^2+9}{x^2-9} = \frac{2x}{x+3}$

b) $\frac{5}{x+1} - \frac{1}{x^2-x-2} = \frac{3}{x-2}$

c) $\frac{x+5}{2x+4} = \frac{x}{x-3} - \frac{2x+9}{x^2-x-6}$

d) $\frac{3x}{2x+3} + \frac{20}{2x^2-x-6} = \frac{4}{x-2}$

5. Solve each rational equation. Round answers to the nearest hundredth.

a) $\frac{x}{2x+3} + \frac{5}{8x+12} = \frac{x+1}{5}$

b) $\frac{x-1}{x} - \frac{(x+2)}{x^2} = \frac{x+1}{3x}$

c) $\frac{3}{2x-3} + \frac{2}{3x+2} = \frac{1}{x}$

d) $\frac{x-5}{2x+10} - \frac{8}{x^2-25} = \frac{x}{x-5}$

6. When solving the following rational equation $\frac{5x}{2x+1} + \frac{x}{x-1} = \frac{3}{2x^2-x-1}$, Petra determines that $x = 1$.

a) Without solving the equation, do you agree with Petra's solution? Explain.

7. The sum of two integers is 12. The difference in their reciprocals is $\frac{2}{9}$. Determine the two integers.

Answers: #1a) 2 b) 3/2 c) 9/2 d) 1/6, -2
e) ¾ , -2 f) -6
#2a) -3/4 b) -3/5 c) 9/2, 1 d) 0
#3a) -2 b) -5 c) -4/7 d) -1/2, 2
#4a) -1 b) 7 c) -1 d) 4, 2/3
#6a) no
#7. 3 and 9 or -6 and 18