5.1 Radicals

$$r\sqrt[n]{x}$$

"r" is the _____

"n" is the _____

"x" is the _____

Perfect Squares:

- 1, 4,
- x^2, x^4,

Perfect Cubes

- 1, 8,
- x^3, x^6,

Changing Mixed Radicals to Entire Radicals

1) $7\sqrt{2}$ 2) $a^4\sqrt{a}$ 3) 5b 4) $4\sqrt{3}$ 5) $j^3\sqrt{j}$

Radicals in Simplest Form

- the radicand contains no factor that is a perfect square, cube, etc.
- there is no radical in the denominator of a fraction.

example: Write as a mixed radical in simplest form.

1) $\sqrt{52}$ 2) $\sqrt{700}$ 3) $5\sqrt{72}$ 4) $\sqrt{50x^2}$

5) $\sqrt{48y^5}$ 6) $\sqrt{63n^7p^4}$ 7) $\sqrt{27x^4y^{12}}$ 8) $\sqrt[4]{m^7}$

Adding/Subtracting Radicals – simplify then combine like terms (terms that have the same index and the same radicand)

example: 1) $\sqrt{5}-6\sqrt{5}$

2) $3\sqrt{5}+2\sqrt{2}-8\sqrt{5}-6\sqrt{2}$

3) $\sqrt{50}+3\sqrt{2}$

4) $-\sqrt{27}+3\sqrt{5}-\sqrt{80}-2\sqrt{12}$

5) $\sqrt{4c}-4\sqrt{9c}$ (c≥0)

Restrictions – if a radical represents a real number and has an ***even index***, the radicand must be non-negative.

example: Given $\sqrt{4-x}$, then $4-x \geq 0$

Note: If the index is ***odd***, the radicand can be any real number.

example: $\sqrt[3]{-27}=$

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5.2 Multiplying and Dividing Radical Expressions

Multiplying Radical Expressions

- multiply the _____ and multiply the _____ (if they have the ***same index***)
- radicals should be simplified before multiplying
- answer in simplest form
- state restrictions for variables (if index is even, the radicand must be _____)

Examples – Multiply and simplify

1) $(5\sqrt{2})(3\sqrt{5})$

2) $(3\sqrt{6})(-4\sqrt{2})$

To Multiply: Multiply the coefficients
Multiply the radicals
(Must have the same index.)
Simplify

3) $7\sqrt{3}(2\sqrt{3}-5\sqrt{7})$

4) $5\sqrt[3]{9}(4\sqrt[3]{2}+9\sqrt[3]{3})$

Recall Distributive Property :

$2(x+4)$
$2x+8$

5) $\left(4\sqrt{2}+2\sqrt{3}\right)\left(5\sqrt{2}-6\sqrt{3}\right)$

6) $\left(\sqrt{5}-3\sqrt{3}\right)\left(2\sqrt{6}+\sqrt{2}\right)$

Recall Double Distributive Property OR **FOIL**:

$(x+2)(x-4)$
$x^2-4x+2x-8$
x^2-2x-8

7) $\left(2\sqrt{5}-3\sqrt{2}\right)^2$

8) $\left(5\sqrt{2}-6\sqrt{3}\right)\left(5\sqrt{2}+6\sqrt{3}\right)$

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5.2 (con't) Dividing Radical Expressions

- divide the _____ and divide the _____ (if they have the ***same index***)
- A rational in simplest form does ***not*** have a radical in the denominator. If necessary, ***rationalize*** the denominator:

  a) monomial denominator – multiply the numerator and denominator by an expression that produces a rational number in the denominator

  b) binomial denominator – multiply the numerator and denominator by the _____ of the denominator. Conjugate: *(a + b)* and *(a – b)* are conjugates

Examples – Find the conjugate of the following

1) $5\sqrt{2}-\sqrt{3} \rightarrow$

2) $-2\sqrt{6}+5\sqrt{7} \rightarrow$

Examples – Divide and simplify

1) $\dfrac{\sqrt{24x^3}}{\sqrt{3x}}$; $x \geq 0$

2) $\dfrac{12\sqrt{6}}{15\sqrt{3}}$

3) $\dfrac{8\sqrt{5}}{4\sqrt{2}}$

4) $\dfrac{7\sqrt{3}}{3\sqrt{x}}$

5) $\dfrac{6}{3+\sqrt{2}}$

6) $\dfrac{5\sqrt{2}}{3\sqrt{2}-\sqrt{3}}$

7) $\dfrac{4+\sqrt{2}}{\sqrt{3}+5\sqrt{2}}$

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5.2 Additional Questions

A. Multiply and simplify

1. $\left(6\sqrt{5}\right)\left(2\sqrt{3}\right)$

2. $\left(\sqrt{27x^5}\right)\left(\sqrt{3x^7}\right)$

3. $\sqrt{10}\left(2\sqrt{10}+\sqrt{5}\right)$

4. $3\sqrt{x}\left(2\sqrt{x}-\sqrt{2}\right)$

5. $\left(\sqrt{x}+1\right)\left(\sqrt{x}-3\right)$

6. $\left(4\sqrt{3}+3\sqrt{5}\right)^2$

Answers: 1. $12\sqrt{15}$ 2. $9x^6$ 3. $20+5\sqrt{2}$ 4. $6x-3\sqrt{2x}$

5. $x-2\sqrt{x}-3$ 6. $93+24\sqrt{15}$ 7. 2 8. $\sqrt{6}$ 9. $\frac{3}{x^2}$ 10. $\frac{\sqrt{30}}{3}$

11. $\frac{4\sqrt{30}}{5}$ 12. $\frac{3-\sqrt{3}}{3}$ 13. $\frac{2+2\sqrt{3}}{3}$ 14. $2\sqrt{3}-2$ 15. $\frac{8\sqrt{3}+2\sqrt{10}-12-\sqrt{30}}{19}$

B. Rationalize each denominator. Express each radical in simplest form.

7. $\dfrac{\sqrt{20}}{\sqrt{5}}$

8. $\dfrac{\sqrt{90}}{\sqrt{15}}$

9. $\dfrac{\sqrt{45x^3}}{\sqrt{5x^7}}$

10. $\dfrac{\sqrt{10}}{\sqrt{3}}$

11. $\dfrac{8\sqrt{15}}{5\sqrt{2}}$

12. $\dfrac{\sqrt{3}-1}{\sqrt{3}}$

13. $\dfrac{6\sqrt{2}+2\sqrt{6}}{3\sqrt{6}}$

14. $\dfrac{4}{\sqrt{3}+1}$

15. $\dfrac{2\sqrt{2}-\sqrt{6}}{2\sqrt{6}-\sqrt{5}}$

5.3 Radical Equations

A. Solving Equations with One Radical Term

Steps: 1. Isolate the radical

2. Square both sides of the equation

3. Solve

4. Check – very important!! Does it meet the restrictions? Is it extraneous?

Example:

a) Solve. State the restrictions on x.

$$5+\sqrt{2x-1}=12$$

b) Solve. State the restrictions on m if $m-\sqrt{2m+3}=6$

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5.3 Radical Equations (con't)

B. Solving Equations with 2 Radicals

Steps:
1. Isolate one radical (the more complex one)
2. Square both sides
3. If necessary, isolate radical again and square both sides again.
4. Solve
5. Check

Example: Solve $\sqrt{z+5}=\sqrt{2z-1}$

Example: Solve $7+\sqrt{3x}=\sqrt{5x+4}+5$

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