Unit 4 – Counting Principles

Fundamental Counting Principle:

- If one task can be performed in a ways AND another task can be performed in b ways, then both tasks can be performed in $a \cdot b$ ways.
- For OR situations, the Fundamental Counting Principle does not apply.

Example 1) Hannah places on her school soccer team. The soccer uniform has three different sweaters, red, white and black. The uniform has three different shorts, red, white and black.

How many different variations of the soccer uniforms can the coach choose from for each game?

Example 2) Kim is choosing a new cellphone. She can choose the 300, 400 or 500 model, in pink, green, indigo, orange or taupe trim. Kim is also deciding whether or not to get a data plan for the phone.

a) How many different choices does Kim have altogether, accounting for model, colour and data plan or not?

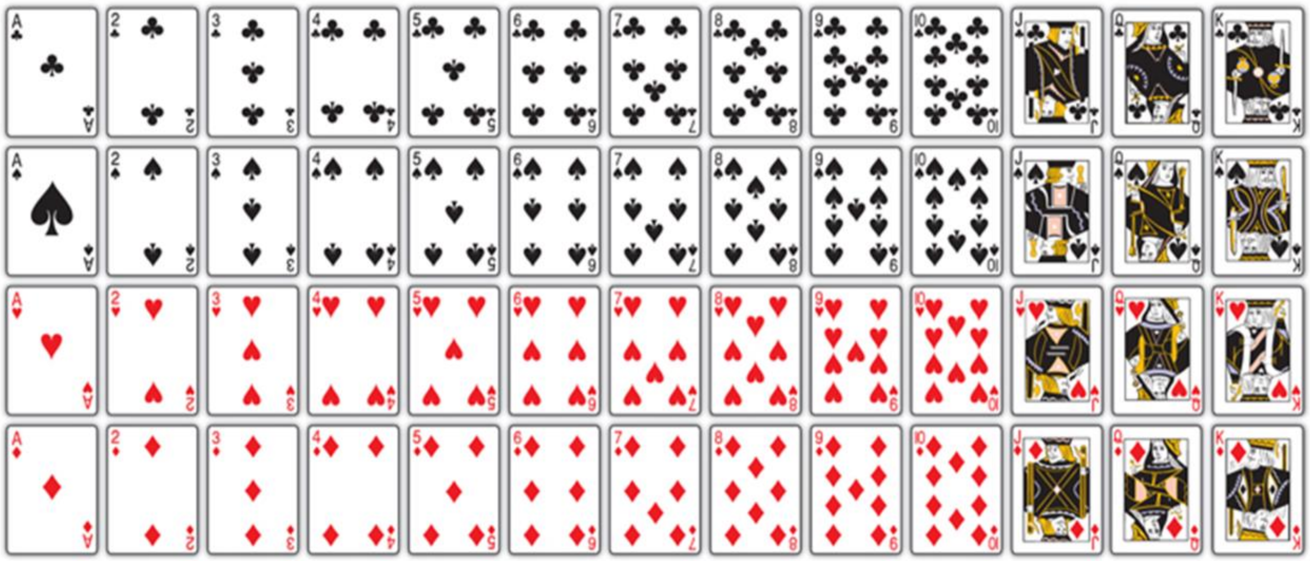
b) The store agent advises Kim not to choose the 300 model if she wants the data plan. Also, the 500 model is not available in indigo or taupe trim. How many choices does Kim have now?

Example 3) A luggage lock opens with the correct three digit code. Each wheel rotates through the digits 0 to 9.

a) How many different three digit codes are possible?

b) Suppose each digit can only be used once in the code. How many different codes are possible when repetition is not allowed?

Example 4) A standard deck of 52 playing cards is shown below:



Count the number of possibilities of drawing a single card and getting:

a) either a black face card or an ace.

b) either a red card or a 10.

c) either a king or a queen

d) either a red card or a spade.

Deck of Cards Cheat Sheet

- **52 cards**
- **26 red, 26 black**
- **4 suits (hearts, diamonds, spades, clubs)**
- **13 of each suit**
- **4 of each card (3 through A)**
- **12 face cards (3 of each suit, J, Q, K)**

Assignment Pg 235 #4 - 6, 8, 9

4.2 – Introducing Permutations and Factorial Notation

How many different single-file lines are possible when Melissa, Nika, and François line up at a cashier at a fast-food restaurant?

Permutation: An arrangement of ________________________ objects in a definite order.
Example: The objects a and b only have two permutations. Either ab or ba.

Factorial Notation: The _____________________ of consecutive _____________________ natural numbers.
n! = (n)(n-1)(n-2)...(3)(2)(1)

4! = 4•3•2•1

Naomi volunteers after school a daycare centre in Whitehorse, Yukon. Each afternoon, around 4 pm, she lines up her group of children at the water fountain to get a drink of water. How many different arrangements of children can Naomi create for the lineup for the water fountain if there are six children in her group?

Factorial Notation

Example 1) Evaluate the following:

a) $10!$ b) $\frac{12!}{9!3!}$ c) $8 \cdot 7!$ d) $\frac{4!}{0!}$

Example 2) Simplify, where $n \in N$.

a) $(n+3)(n+2)!$ b) $\frac{(n+1)!}{(n-1)!}$ c) $\frac{n}{n!}$

Example 3) Solve $\frac{n!}{(n-2)!} = 90, where\ n \in I$

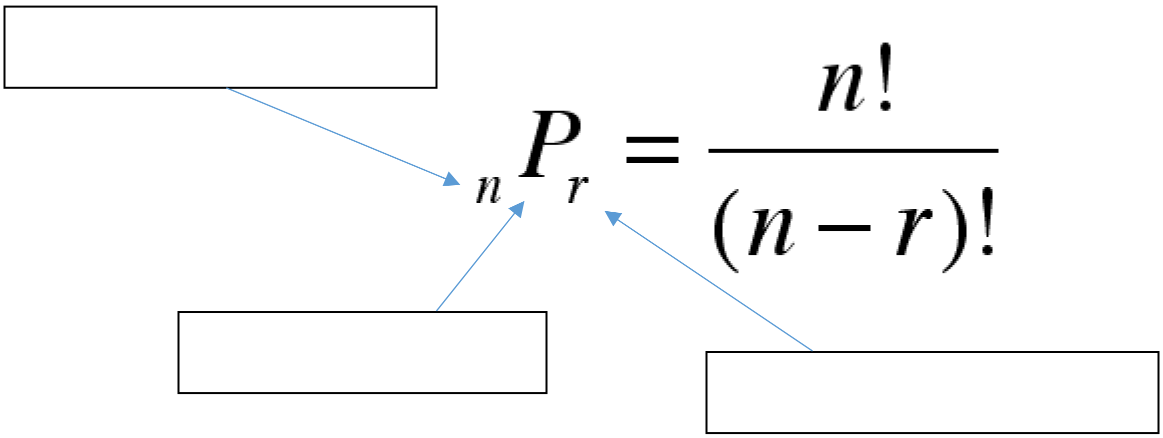
Example 4) Solve $\frac{(n+4)!}{(n+2)!} = 6, where\ n \in I$

Assignment Pg 243 #1, 3, 4 - 6, 11

4.3 – When All Objects are Distinguishable

How many 3 letter permutations can you make with the letters in the word MATH?

Permutation Formula



$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Example 1) Matt has downloaded 10 new songs from an online music store. He wants to create a playlist using 6 of these songs arranged in any order. How many different 6 song playlists can be created from his new downloaded songs?

Determine all the possible 7-song, 8-song and 9-song playlists. What happens to the value as *r* approaches *n*?

Note!

0! = 1, you can think of this as there only being one way to arrange an empty set.

Therefore, any algebraic expression that involves factorials is defined as long as the expression is greater than or equal to zero.

$(n + 4)!$ is only defined for $n \leq -4$ and $n \in I$.

Example 2) State the values of *n* for which each expression is defined, where $n \in I$.

a) $(n+3)!$

b) $\frac{n!}{(n+2)!}$

Example 3) Tania needs to create a password for a social networking site. The password can use any of the digits from 0 to 9, and or any letters of the alphabet. The password is case sensitive, so she can use both lower and uppercase letters. A password must be at least 5 characters, to a maximum of 7 characters. Each character can only be used once.

a) Determine the total number of possible passwords.

b) Suppose another site requires a password of exactly 8 letters using the same character requirements. How many possible passwords are there now? Is this site more or less secure than the first one?

Example 4) At a used car lot, 7 different cars close to the street for easy viewing.

a) The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

b) The three red cars must be parked side by side. How many ways can the seven cars be parked?

c) The three red cars must be parked side by side and the other four cars must be parked side by side. How many ways can the seven cars be parked?

Example 5) A social insurance number (SIN) in Canada consists of a 9 digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position of the number, how many SINs can be created if each digit can be repeated? How does this compare with the number of SINs that can be created if no repetition is allowed?

In reality, the Canadian government does not use 0, 8 or 9 as the first digit when assigning SINs to citizens, and repetition of digits is allowed. How many nine digit SINs do not start with 0, 8 or 9?

SINs starting with the digit 9 are issued to temporary residents. How many SINs are there for temporary residents?

Assignment Pg 255 #1, 6, 8, 9, 11

4.4 – Permutations When Objects are Identical

Tom is putting away the dishes after supper. He has to stack seven plates. Three of them are white and identical, while the remaining four plates are red, green, yellow and blue.

A. If all seven plates are different, how many ways could be stack the plates?

B. Tom decided to think about the plates as if they were different. To do this, he represented the three identical white plates using 3 different letter codes: W_1, W_2, W_3. He then used R for the red plate, G for the green plate, Y for the yellow plate, and B for the blue plate. List all the ways the plates can be stacked if the white plates are stacked on top of the four colored plates (in the order red, green, yellow, blue).

C. Examine your list in part B. Recognizing that the white plates are identical, how many arrangements in your list are really the same arrangement? How does this number relate to the number of white plates?

D. Use your answers for parts A and C to write an expression that represents the number of different ways these seven plates can be stacked. Use your expression to calculate the number.

E. Suppose Tom had to stack these plates.

Write an expression to represent the number of different ways to stack the plates.

F. Suppose Tom had to stack these plates.

Write an expression to represent the number of different ways to stack the plates.

G. The next time Tom puts away the dishes, the number of arrangements that are possible when he stacks the plates is represented by the expression:

$$\frac{10!}{2!\,3!\,4!}$$

How many plates will be he be stacking and what colours might they be?

Permutations when some objects are identical:

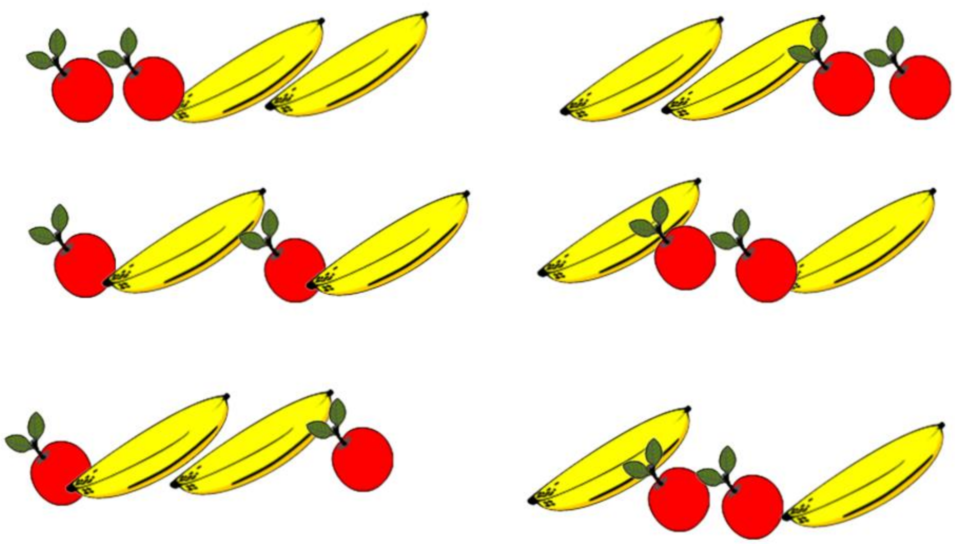
There are fewer permutations when some objects are identical, as some of the arrangements would be identical.

The number of permutations of n objects, where a are identical, another b are identical, another c are identical, and so on, is:

Example 1) In the set of four objects, apple, apple, banana, banana, the number of different permutations, P, is:

$$P = \frac{4!}{2!2!}$$

The six possible permutations are:



Example 2) In the mountainous regions of India, China, Nepal and Bhutan, it is common to see prayer flags. Each flag has a prayer written on it, and colour is used to symbolize different elements: green (water), yellow (earth), white (air/wind), blue (sky/space), and red (fire). How many different arrangements of the same prayer can Dorji make using these 9 flags: 1 green, 1 yellow, 2 white, 3 blue and 2 red?

Example 3) How many ways can the letters of the word CANADA be arranged, if the first letter must be an N and the last letter must be a C?

What is there are no conditions?

What if the first letter has to be a C?

Assignment Pg 267 #1-8

4.5 and 4.6 – Combinations

Combination:

Example: Two objects, a and b have one combination because ab is the same as ba.

$$_nC_r = \frac{n!}{r!(n - r)!}$$

n is the number of objects

r is the number in the group

The school athletic council has five members: Jill, Ted, Rhaj, Yvette and Martin. This Wednesday, they plan to hold a bake sale. How can you count the number of committees of at least 2 people that can be chosen to sell baked goods during lunch?

Example 1) Five cards are dealt to each person in a card game. How many ways can you be dealt a hand that has only red cards?

Red cards in the deck? C Cards in your hand?

Example 2) Each year during the Festival du Voyageur, held during February in Winnipeg, Manitoba, high school students compete in the Voyageur Snow Sculpture Contest. This year, Amir's school will enter a three person team. Nine students have volunteered to be on the team. In how many ways can a team of three snow sculptors be chosen to represent Amir's school from the nine students who have volunteered?

Example 3) A restaurant serves 10 flavours of ice cream. Danielle has ordered a large sundae with three scoops of ice cream. How many different ice cream combinations does Danielle have to choose from, if she wants each scoop to be a different flavour?

What if her favorite flavour is chocolate, and one scoop must be chocolate. How many combinations of ice cream are possible?

Example 4) Tanya is the coach of a Pole Push team that consists of nine players: five male and four female. In each competition, teams of four compete against each other to push their competitors out of a circle. The team that is successful wins.

a) How many different four-person teams does Tanya have to choose from for an all-male competition?

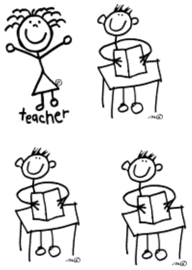
b) How many different four-person teams does Tanya have to choose from, with two male and two female competitors for a mixed competition?

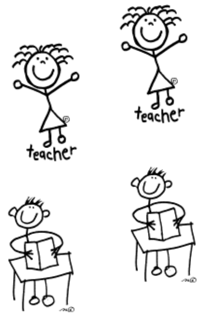
c) How many different four-person teams does Tanya have to choose from that has at least one female?

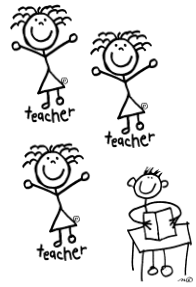
CASES

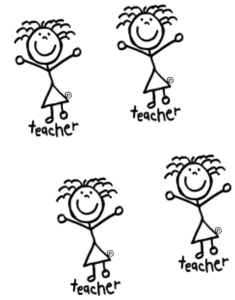
MULTIPLY within each case, and ADD cases together

Example 5) A planning committee is to be formed for a school-wide Earth Day program. There are 13 volunteers: 8 teachers and 5 students. How many ways can the principle choose from a 4 person committee that has at least one teacher?









__________ + __________ + __________ + __________ =

Assignment Pg 280 #4 - 8, 11

4.7 – Solving Counting Problems

Fundamental Counting Principle	**Permutations**	**Combinations**
a ways AND *b* ways = *ab* ways	Order Matters ($_nP_r$)	Order Does NOT Matter ($_nC_r$)
Examples: • SIN numbers • Combination Locks • Lining up for a picture	Examples: • Finishing 1st, 2nd and 3rd • Committees with positions like President, Secretary, Treasurer	Examples: • Finishing in the top 3 • Committees without special positions • Hands of cards

CASES

MULTIPLY within each case, and ADD cases together

There are five swimmers in the first heat of a race: Aubrey, Betz, Cam, Deanna, and Elena.
a) How many ways can the five swimmers finish first, second, and third?

FCP

P

C

b) How many ways can the five swimmers qualify for the final race if the top three finishers qualify?

Example 1) A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teachers sit in a row of nine chairs for this pose?

FCP

P

C

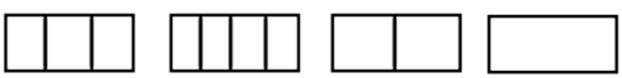
For another pose, the photographer wants to two tallest students, Jill and Sam, to sit at either end, Jill on the left and Sam on the right, and the teacher in the middle. How many different arrangements are there for this pose?

Example 2) Combination problems are common in computer science. Suppose there is a set of 10 different data items to be represented by {a, b, c, d, e, f, g, h, i, j} to be placed into four different memory cells in a computer. Only 3 data items are to be placed in the first cell, 4 data items in the second cell, 2 data items in the third cell, and 1 data item in the last cell. How many ways can the 10 data items be placed in the four memory cells?

FCP

P

C



Example 3) How many different five-card hands that contain at most one black card can be dealt to one person from a standard deck of playing cards?

Assignment Pg 288 #1-8