

Answers

ANSWERS 1.1

1. $5! = 120$
2. $\frac{5!}{2! \cdot 2!} = 30$
3. $\frac{1 \cdot 7!}{4! \cdot 2!} = 105$
4.
 - (a) $5 \cdot 5 \cdot 5 = 125$
 - (b) $5 \cdot 4 \cdot 3 = 60$
5. $21 \cdot 21 \cdot 21 \cdot 9 \cdot 10 \cdot 10 = 8\,334\,900$
6.
 - (a) $6 \cdot 7 \cdot 7 = 294$
 - (b) $6 \cdot 7 \cdot 3 = 126$
 - (c) $6 \cdot 7 \cdot 2 = 84$
7.
 - (a) $6 \cdot 6 \cdot 5 = 180$
 - (b) $5 \cdot 5 \cdot 3 = 75$ (First choose the last digit.)
 - (c) $5 \cdot 5 \cdot 1 = 25$ end in 5 and $6 \cdot 5 \cdot 1 = 30$ end in 0 for a total of 55.
8. $(6-1)! = 120$
9. $\frac{(7-1)!}{2} = 360$
10. ${}^7C_4 = 35$
11. ${}_{10}C_2 = 45$ (It takes a set of 2 teams to make a game.)
12. ${}_9C_3 = 84$ (It takes a set of 3 points to form a triangle.)
13. $\frac{11!}{2! \cdot 7! \cdot 2!} = 1980$
14. $6(3!)(5!) = 4320$ (The girls can be together in any one of 6 different clusters. Within each cluster, they can be arranged in $3!$ ways. The boys can be arranged in the remaining 4 positions in $4!$ ways.)
15. $1(2!)(3!) = 12$ (Person A must be seated. Then B and C can be arranged about A in $2!$ ways. The remaining 3 people can be arranged in $3!$ ways.)
16. ${}_4C_3 \cdot {}_4C_1 + {}_4C_4 \cdot {}_4C_0 = 17$ (There could be 3 girls and 1 boy or 4 girls and 0 boys.)
17.
 - (a) ${}_7C_3 = 35$
 - (b) ${}_6C_2 \cdot {}_4C_1 = 60$
 - (c) ${}_{10}C_3 = 120$
18.
 - (a) ${}_{26}C_{10} = 5\,311\,735$
 - (b) ${}_{26}C_5 \cdot {}_{26}C_5 = 4\,327\,008\,400$
 - (c) ${}_{39}C_{10} = 635\,745\,396$
 - (d) ${}_{13}C_4 \cdot {}_{13}C_4 \cdot {}_2C_2 = 39\,875\,550$
 - (e) ${}_{40}C_{10} = 847\,660\,528$
19. ${}_6C_3 \cdot {}_3C_2 \cdot 5! = 7200$
20.
 - (a) ${}_4C_3 \cdot {}_4C_2 \cdot 5! = 2880$
 - (b) ${}_1C_1 \cdot {}_3C_2 \cdot {}_4C_2 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 432$
21. ${}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5 = 31$ (You can leave a set of any 1 of 5 coins or a set of any 2 of 5 coins etc.)
22. We can make $4 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 24$ arrangements that contain no C's. We can make $1 \cdot 1 \cdot {}_4C_3 \cdot 4 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 96$ arrangements that contain a C, a B in the middle position and 3 more letters from {U, I, A, L}. We can make ${}_2C_2 \cdot 1 \cdot {}_4C_2 \cdot \frac{4 \cdot 3 \cdot 1 \cdot 2 \cdot 1}{2!} = 72$ arrangements that contain 2 C's, and a B in the middle position. Thus there is a total of 192 arrangements.
23. Certainly they were not out of numbers. With only 1 million people in the province, each person would have to be driving 8 vehicles (including children). Clearly it was a decision to indicate that a change in the license plate had been made. Everyone with digits first has the logo.
24. There are $\frac{12!}{3! \cdot 3! \cdot 3! \cdot 3!} = 369\,600$ different routes. It would take $\frac{369\,600}{365}$ or 1012.6 years. You would never be more than 6 blocks from your house.